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THE FEBRUARY MEETING IN NEW YORK

The two hundred eighty-seventh meeting of the American Mathematical Society was held at Columbia University, on Saturday, February 28, 1931, extending through the usual morning and afternoon sessions. The attendance included the following ninety-five members:

C. R. Adams, R. B. Adams, R. P. Agnew, A. A. Albert, R. L. Anderson, R. C. Archibald, H. E. Arnold, F. W. Beal, A. A. Bennett, T. C. Benton, A. C. Berry, A. B. Brown, J. H. Bushey, S. S. Cairns, W. D. Cairns, B. H. Camp, G. A. Campbell, Alonzo Church, L. W. Cohen, J. L. Coolidge, E. H. Cutler, M. D. Darkow, F. W. Doermann, J. L. Dorroh, L. P. Eisenhart, J. M. Feld, T. S. Fiske, W. B. Fite, W. W. Flexner, Tomlinson Fort, R. M. Foster, T. C. Fry, J. W. Glover, M. C. Gray, F. C. Hall, S. B. Hansell, L. A. Hazeltine, Robert Henderson, Einar Hille, J. C. Hughes, W. A. Hurwitz, L. W. Hussey, M. H. Ingraham, S. A. Joffe, R. A. Johnson, Edward Kasner, L. S. Kennison, M. V. Kenny, J. R. Kline, M. S. Knebelman, E. A. Knobelauch, J. H. Kusner, A. W. Landers, C. A. Lovell, N. H. McCoy, A. E. Meder, F. H. Miller, A. K. Mitchell, L. T. Moore, Marston Morse, G. W. Mullins, C. A. Nelson, Oystein Ore, L. J. Paradiso, W. O. Pennell, E. L. Post, H. W. Raudenbush, H. W. Reddick, R. G. D. Richardson, J. F. Ritt, H. P. Robertson, Irwin Roman, H. M. Schlauch, C. E. Seely, Wladimir Seidel, Stefan Serghiesco, L. P. Siceloff, L. G. Simons, D. E. Smith, A. H. Sprague, Paul Szabo, J. D. Tamarkin, J. M. Thomas, Oswald Veblen, J. L. Walsh, J. H. M. Wedderburn, Louis Weisner, M. E. Wells, A. P. Wheeler, H. S. White, L. E. Widmark, Norbert Wiener, W. L. G. Williams, W. A. Wilson, J. W. Young.

At the meeting of the Council, held Friday evening, February 27, at the Governor Clinton Hotel, the following persons were elected to membership in the Society:

Dr. David Gordon Bourgin, University of Illinois;
Mr. Jack Britton, University of Colorado;
Miss Lillian Dundes, Columbia University;
Mr. Jules Alphonse Larrivée, Massachusetts Institute of Technology;
Mr. Simon Lehrman, physical chemist, Hamilton, Canada;
Mr. Charles Bradfield Morrey, Jr., Harvard University;
Mr. Edwin J. Purcell, Boulder, Colorado;
Dr. Wladimir Seidel, Harvard University;
Dr. George William Starcher, Ohio University;
Professor Hugh E. Stelson, Kent State College, Kent, Ohio;
Mr. Albert William Tucker, Princeton University;
Dr. Wilfrid Wilson, University of Illinois.

As nominees of Allyn and Bacon:

Mr. Gaines Barrett Lang, University of Illinois;
Mr. Edgar Raymond Lorch, Columbia University;
Mr. Carroll Vincent Newsom, University of Michigan.

As nominees of the National Life Insurance Company of the United States of America:

Mr. David Joseph Colbert, Brown University.
Mr. Vivian S. Lawrence, Jr., Cornell University;
Dr. Clifford William Mendel, University of Chicago;
Mr. Herbert Earl Spencer, Cornell University.

As nominee of Members of the Department of Mathematics of the University of Wisconsin:

Mr. Nathan Schwid, University of Wisconsin.

It was announced that Professor P. W. Bridgman, of Harvard University, is to give the ninth Josiah Willard Gibbs Lecture in December, 1931.

At the meeting of the Council there was a prolonged discussion of the financial needs of the Society, and the Committee on Funds was requested to investigate further.

At the beginning of the afternoon session, by invitation of the Program Committee, Professor C. R. Adams delivered an address entitled *Linear q-difference equations*. This address will appear in full in a later issue of this Bulletin.

President Eisenhart presided at both sessions, relieved by Professor H. S. White.

Titles and cross-references* to the abstracts of the papers read at this meeting follow below. The papers whose abstract numbers are followed by the letter *t* were read by title. Dr. Cauer was introduced by Professor Fort, Dr. Harshbarger by Professor Sharpe, and Dr. Seidel by Professor Walsh.

1. *On the approximation of continuous functions by linear combinations of continuous functions*, by Dr. Wladimir Seidel (National Research Fellow). (Abstract No. 37–3–115.)

2. *An interpolation problem of functions with positive real part*, by Dr. W. Cauer. (Abstract No 37–3–116.)

3. *On interpolation and approximation by rational functions with preassigned poles*, by Professor J. L. Walsh. (Abstract No. 37–3–117.)

* See this Bulletin, vol. 36, p. 6, footnote.

4. *On convergence and summability of double orthogonal series*, by Dr. R. P. Agnew (National Research Fellow). (Abstract No. 37–1–105.)

5. *Reflections in function space*, by Dr. L. S. Kennison. (Abstract No. 37–3–118.)

6. *Quotients of differential forms*, by Professor J. F. Ritt. (Abstract No. 37–3–119.)

7. *Representation by Fourier series of functions whose successive intervals lie in separate planes differing by a phase angle*, by Mr. W. O. Pennell. (Abstract No. 37–3–120.)

8. *On repeat operations*. Preliminary report, by Mr. L. E. Widmark. (Abstract No. 37–3–121.)

9. *On the join of two complexes*, by Dr. A. B. Brown. (Abstract No. 37–1–106.)

10. *On continuous curves which are homogeneous except for a countable infinity of points*, by Dr. T. C. Benton. (Abstract No. 37–3–122.)

11. *A classification of all pure Riemann matrices of genus* $p \leqq 8$, by Dr. A. A. Albert. (Abstract No. 37–3–123–*t*.)

12. *A Galois theory in semi-simple rings*, by Dr. Jakob Levitzki. (Abstract No. 37–1–112–*t*.)

13. *On the Bring-Jerrard quintic*, by Professor Raymond Garver. (Abstract No. 37–3–124–*t*.)

14. *Tschirnhaus transformations and the real roots of an equation*, by Professor Raymond Garver. (Abstract No. 37–3–125–*t*.)

15. *On the curvatures of a curve in Riemann space*, by Dr. E. H. Cutler. (Abstract No. 37–3–126–*t*.)

16. *Penosculating conics of a plane curve*, by Professor J. W. Lasley. (Abstract No. 37–3–127–*t*.)

17. *On the derivatives of harmonic functions on the boundary*, by Professor O. D. Kellogg. (Abstract No. 37–1–113–*t*.)

18. *On convex arcs and surfaces*, by Dr. S. B. Littauer (National Research Fellow). (Abstract No. 37–3–128–*t*.)

19. *On spaces* V_ω, by Professor R. G. Putnam. (Abstract No. 37–3–129–*t*.)

20. *On homeomorphic graphs and the connectivity of graphs*, by Mr. Hassler Whitney. (Abstract No. 37–3–130–*t*.)

21. *On the cyclic connectivity theorem*, by Dr. G. T. Whyburn. (Abstract No. 37–1–109–*t*.)

22. *On semi-metric spaces*, by Professor W. A. Wilson. (Abstract No. 37–1–110–*t*.)

23. *Invariance under contact transformation*, by Professor Edward Kasner. (Abstract No. 37–3–131.)

24. *On the Frenet formulas for a general subspace of a Riemann space*, by Dr. E. H. Cutler. (Abstract No. 37–3–132.)

25. *Linear equations in non-commutative domains*, by Professor Oystein Ore. (Abstract No. 37–1–108.)

26. *The integers represented by sets of positive ternary quadratic forms*, by Dr. A. A. Albert. (Abstract No. 37–3–133.)

27. *On nilpotent subrings*, by Dr. Jakob Levitzki. (Abstract No. 37–1–111.)

28. *On the resultant of three double binary forms*, by Dr. N. H. McCoy (National Research Fellow). (Abstract No. 37–1–107.)

29. *The geometric configuration defined by a special algebraic relation of genus* 4, by Dr. Frances Harshbarger. (Abstract No. 37–3–134–*t*.)

TOMLINSON FORT,
Associate Secretary

THE APRIL MEETING IN NEW YORK

The two hundred eighty-eighth meeting of the American Mathematical Society was held at Columbia University, on Friday and Saturday, April 3–4, 1931. The attendance included the following one hundred forty-one members of the Society:

C. R. Adams, R. B. Adams, R. P. Agnew, A. A. Albert, R. A. Arms, N. H. Ball, T. C. Benton, Wilhelm Blaschke, H. F. Blichfeldt, Bertha Boschwitz, Paul Bradt, A. B. Brown, B. L. Brown, L. H. Bunyan, S. S. Cairns, R. H. Cameron, E. T. Carroll, G. G. Chambers, Alonzo Church, W. R. Church, R. P. Conkling, H. R. Cooley, L. P. Copeland, E. H. Cutler, Tobias Dantzig, Edgar Dehn, E. C. Doak, H. L. Dorwart, Jesse Douglas, Lillian Dundes, L. P. Eisenhart, Arne Fisher, T. S. Fiske, W. B. Fite, Edward Fleisher, W. W. Flexner, Tomlinson Fort, A. L. Foster, R. M. Foster, Philip Franklin, Orrin Frink, K. G. Fuller, B. P. Gill, D. C. Gillespie, Eli Gourin, H. S. Grant, C. H. Graves, C. C. Grove, F. C. Hall, G. A. Hedlund, L. S. Hill, Einar Hille, A. A. Himwich, Lulu Hofmann, T. R. Hollcroft, W. G. Hubert, J. C. Hughes, R. L. Jeffery, S. A. Joffe, R. A. Johnson, F. E. Johnston, E. G. Keller, L. S. Kennison, E. A. Kholodovsky, B. F. Kimball, M. S. Knebelman, P. A. Knedler, Mark Kormes, A. M. M. Lehr, S. A. Lepeshkin, Jakob Levitzki, S. B. Littauer, Marie Litzinger, E. O. McCormick, E. J. McShane, L. A. MacColl, W. A. Manning, A. E. Meder, F. H. Miller, H. H. Mitchell, Morris Monsky, L. T. Moore, Richard Morris, David Moskovitz, C. A. Nelson, Philip Newman, C. O. Oakley, Oystein Ore, F. W. Owens, L. J. Paradiso, J. K. Peterson, G. A. Pfeiffer, H. R. Phalen, Hillel Poritsky, E. L. Post, R. G. Putnam, H. W. Reddick, L. H. Rice, R. G. D. Richardson, J. F. Ritt, H. P. Robertson, Robin Robinson, H. A. Ruger, S. A. Schelkunoff, H. M. Schlauch, J. A. Schouten, C. H. W. Sedgwick, C. E. Seely, Wladimir Seidel, Casper Shanok, I. M. Sheffer, L. P. Siceloff, L. L. Silverman, L. G. Simons, James Singer, L. L. Smail, C. E. Smith, P. A. Smith, A. H. Sprague, M. E. Stark, D. J. Struik, J. D. Tamarkin, J. H. Taylor, J. M. Thomas, T. Y. Thomas, J. E. Thompson, C. C. Torrance, A. W. Tucker, Annita Tuller, R. N. Van Arnam, J. L. Vanderslice, Oswald Veblen, Louis Weisner, A. P. Wheeler, R. A. Whelan, H. S. White, G. T. Whyburn, D. V. Widder, W. A. Wilson, H. M. Wright, Oscar Zariski.

A meeting of the Council of the Society was held on the evening of April 3, and various items of business transacted were announced to the Society.

The Secretary announced the election of the following nine persons to membership in the Society:

Miss Beatrice Aitchison, Johns Hopkins University;
Mr. Edwin Ford Beckenbach, Rice Institute;

Professor Wilhelm Blaschke, University of Hamburg;
Professor Clark M. Cleveland, University of Texas;
Mr. Charles Hewitt Dix, Rice Institute;
Professor Jan A. Schouten, University of Delft;
Mr. Casper Shanok, New Haven;
Professor Ruth W. Stokes, Winthrop College;
Miss Marian A. Wilder, University of Minnesota.

The following appointments were announced: to represent the Society at the inauguration of Harry Woodburn Chase as President of the University of Illinois on May 1, 1931, Vice-President E. P. Lane; as additional members of the Committee on the Cole Prize, Professors R. D. Carmichael and G. H. Hardy.

It was decided to omit the February and April meetings in New York City during 1932 and to have only one meeting, this to be held during the Easter recess on March 25 and 26.

The Council authorized an active campaign to increase the membership in the Society.

On Friday afternoon, Professor J. A. Schouten, of the University of Delft, delivered an address on *A revision of the theory of curvature of a V_m in V_n*. This address was followed by a general session of the Society, at which several papers were read. Saturday morning and part of Saturday afternoon were devoted to a symposium on the theory of groups, at which the following addresses were read: *Linear groups*, by Professor H. F. Blichfeldt, of Stanford University; *The primitive permutation groups*, by Professor W. A. Manning, of Stanford University; *The Schreier theory of continuous groups*, by Professor Wilhelm Blaschke, of the University of Hamburg. The program of the Saturday afternoon session also included an address by Professor Harald Bohr, of the University of Copenhagen, *On Dirichlet series and power series with infinitely many variables*. President Eisenhart presided at these three sessions.

On Friday morning the Society met in two sections for the presentation of short papers. The titles and cross references to the abstracts of the papers read at these sessions, and at the general session on Friday afternoon after the address by Professor Schouten, follow below; papers whose abstract numbers are followed by the letter *t* were read by title. The papers numbered 1 to 5 were read before the section of Analysis, Professor

Tamarkin presiding, those numbered 6 to 20 before the section of Algebra and Geometry, Professor Hollcroft presiding, and those numbered 21 to 23 at the general session. Mr. Myers was introduced by Professor Marston Morse, Mr. Doob by Professor J. L. Walsh, and Professor Vaidyanathaswami by Professor A. J. Kempner.

1. *Volterra integral equations of the first kind*, by Professor R. L. Jeffery. (Abstract No. 37-5-172.)

2. *On analytic functions with infinitely many zeros*, by Dr. Wladimir Seidel (National Research Fellow). (Abstract No. 37-5-173.)

3. *Linear q-difference equations of continuous order.* Preliminary report, by Dr. L. S. Kennison. (Abstract No. 37-5-174.)

4. *Adjoint systems in the problem of Mayer under general end conditions*, by Mr. S. B. Myers. (Abstract No. 37-5-175.)

5. *The boundary values of an analytic function*, by Mr. J. L. Doob. (Abstract No. 37-5-176.)

6. *On plane Cremona triadic characteristics*, by Mr. C. C. Torrance. (Abstract No. 37-3-141.)

7. *On the differential geometry of surfaces in non-euclidean space*, by Dr. Robin Robinson. (Abstract No. 37-5-177.)

8. *On a certain inequality of Steiner*, by Dr. E. J. McShane (National Research Fellow). (Abstract No. 37-5-178.)

9. *Semi-linear equations.* Part I: *Regular polygons*, by Professor C. O. Oakley. (Abstract No. 37-5-179.)

10. *On normal products of algebras*, by Dr. Jakob Levitzki. (Abstract No. 37-5-180.)

11. *On the irregularity of cyclic multiple planes*, by Dr. Oscar Zariski. (Abstract No. 37-5-181.)

12. *On illusory Plückerian characters*, by Dr. Oscar Zariski. (Abstract No. 37-5-182.)

13. *Pfaffians in parametric form*, by Professors C. L. E. Moore and Philip Franklin. (Abstract No. 37-3-136-*t*.)

14. *The geometric interpretation of some elementary formulas of analytic geometry*, by Professor Philip Franklin. (Abstract No. 37-3-139-*t*.)

15. *Regions of positive and negative curvature on closed surfaces*, by Professor Philip Franklin. (Abstract No. 37-3-137-*t*.)

16. *Two functional equations with integral arguments*, by Professor Philip Franklin. (Abstract No. 37-3-138-*t*.)

17. *A geometric theory of solution of linear inequalities*, by Professor Ruth W. Stokes. (Abstract No. 37-3-140-*t*.)

18. *On equivalence of methods of evaluation of sequences*, by Dr. R. P. Agnew (National Research Fellow). (Abstract No. 37-5-183-*t*.)

19. *The theory of multiplicative arithmetic functions*, by Professor R. Vaidyanathaswami. (Abstract No. 37-5-184-*t*.)

20. *The Koebe mapping theorem and the problem of Plateau*, by Professor Jesse Douglas. (Abstract No. 37-5-185-*t*.)

21. *Geodesics of Pfaffians*, by Professors C. L. E. Moore and Philip Franklin. (Abstract No. 37-3-135.)

22. *Systems of algebraic differential equations*, by Professor J. F. Ritt. (Abstract No. 37-5-186.)

23. *Certain types of criteria for irreducibility of polynomials*, by Mr. H. L. Dorwart and Professor Oystein Ore. (Abstract No. 37-5-187.)

TOMLINSON FORT,
Associate Secretary

THE APRIL MEETING IN CHICAGO

The two hundred eighty-ninth regular meeting of the Society was held at the University of Chicago on Friday and Saturday, April 3–4, 1931. About one hundred fifty persons attended the meeting, among whom were the following one hundred twelve members of the Society:

J. V. Atanasoff, F. L. Axen, R. P. Baker, R. H. Bardell, R. W. Barnard, Walter Bartky, M. M. Beenken, S. F. Bibb, G. A. Bliss, J. W. Bower, H. R. Brahana, R. C. Bullock, W. D. Cairns, G. N. Carmichael, R. D. Carmichael, E. W. Chittenden, A. B. Coble, L. W. Cohen. P. T. Copp, Max Coral, L. C. Cox, H. B. Curtis, L. E. Dickson, J. M. Earl, John Eiesland, E. B. Escott, H. P. Evans, H. S. Everett, E. M. Feltges, K. W. Folley, Cornelius Gouwens, G. H. Graves, L. M. Graves, L. W. Griffiths, V. G. Grove, B. L. Hagen, W. L. Hart, G. A. Herr, M. R. Hestenes, E. H. Hildebrandt, F. H. Hodge, T. F. Holgate, D. L. Holl, Mildred Hunt, M. H. Ingraham, Dunham Jackson, M. M. Johnson, H. F. S. Jonah, O. D. Kellogg, P. W. Ketchum, W. S. Kimball, J. M. Kinney, W. C. Krathwohl, E. P. Lane, R. E. Langer, Lincoln LaPaz, C. G. Latimer, M. I. Logsdon, J. V. McKelvey, C. C. MacDuffee, W. D. MacMillan, Wilhelm Maier, H. W. March, Morris Marden, T. E. Mason, E. D. Meacham, E. W. Miller, E. H. Moore, T. W. Moore, Max Morris, E. J. Moulton, A. L. Nelson, R. L. Newlin, E. J. Olson, H. L. Olson, C. I. Palmer, G. A. Parkinson, W. R. Paul, E. D. Pepper, H. A. Perkins, H. P. Pettit, H. H. Pixley, H. S. Pollard, J. E. Powell, G. Y. Rainich, M. S. Rees, W. T. Reid, H. L. Rietz, P. G. Robinson, A. E. Ross, W. E. Roth, R. G. Sanger, Henry Schultz, G. T. Sellew, H. A. Simmons, M. E. Sinclair, H. E. Slaught, I. S. Sokolnikoff, E. B. Stouffer, V. B. Teach, P. L. Trump, H. L. Turrittin, E. B. Van Vleck, J. I. Vass, H. S. Wall, L. E. Ward, Warren Weaver, M. B. White, F. B. Wiley, Y. K. Wong, F. E. Wood, Jacob Yerushalmy.

On Friday afternoon Professor G. Y. Rainich gave a symposium address on *Some trends in mathematical physics*. Professor Graves, who presided at this session, expressed the thanks of those present to the speaker.

On Friday evening members and their guests attended a dinner in the Hotel Windermere West. Professor H. E. Slaught was toastmaster. He called upon Professor Jackson who extended a warm invitation to all present to attend the summer meeting in Minneapolis. Associate Secretary Ingraham spoke of the financial needs of the Society as formulated under the leadership of the Committee on Funds; and Professor W. D. Cairns presented the needs of the Mathematical Association of

America; Professor O. D. Kellogg spoke of the national unity of the Society. One hundred thirteen persons attended this dinner.

The Saturday morning session opened with an address by Professor Dickson on *Proof of Waring's theorem on fifth powers*. This paper will appear in this Bulletin.

The meetings were held in Eckhart Hall, the splendid new mathematics building of the University of Chicago. On Friday afternoon tea was served in the Commons Room, and throughout the meeting this room was available for social purposes. It is in this room that the fine portrait of Professor E. H. Moore is hung.

Professors H. L. Rietz and Warren Weaver presided at the Friday morning session, and Professor Carmichael and Vice-President Lane at the session on Saturday morning.

Dr. Schoenberg and Dr. Young were introduced by Professor G. A. Bliss, Mr. Wilks and Mr. Craig by Professor H. L. Rietz, Mr. Oldenburger by Professor G. Y. Rainich, Mr. King by Professor W. S. Kimball, Dr. Bristow by Professor R. D. Carmichael, Miss Schottenfels by Professor L. E. Dickson, Mr. Case by Professor M. I. Logsdon, Professor Hughes by Professor T. E. Mason, and Mr. Robinson by Professor E. W. Chittenden.

GENERAL SESSION, Friday morning.

1. *Second degree addition theorems for even functions*, by Professor J. D. Grant. (Abstract No. 37-5-189-*t*.)

2. *Convergence criteria for continued fractions*, by Professor H. S. Wall. (Abstract No. 37-3-149-*t*.)

3. *General theorems on the convergence of sequences of Padé approximants*, by Professor H. S. Wall. (Abstract No. 37-5-205.)

4. *Note on geodesics in higher spaces*, by Dr. I. Schoenberg (International Research Fellow). (Abstract No. 37-5-199.)

5. *Integral equation of the* Γ *function*, by Professor Wilhelm Maier. (Abstract No. 37-5-197.)

6. *The Gaussian law of error*, by Professor Warren Weaver. (Abstract No. 37-5-188.)

7. *On the distribution of statistics in samples from a normal universe of two variates with matched sampling of one variable*, by Mr. S. S. Wilks. (Abstract No. 37-3-145.)

8. *The frequency of x/y for correlated variables*, by Mr. A. E. Craig. (Abstract No. 37-5-201.)

9. *Reduced matrices of class* 3, by Mr. Rufus Oldenburger. (Abstract No. 37-5-202.)

10. *The elastic instability of members with thin outstanding flanges of non-isotropic material*, by Professor H. W. March. (Abstract No. 37-5-203.)

11. *Theory of heat conduction and convection from a hot vertical plate*, by Professor W. S. Kimball and Mr. W. J. King. (Abstract No. 37-5-206.)

12. *Automatic integration and electrical computing instruments*, by Dr. A. E. Young. (Abstract No. 37-5-192.)

13. *On the necessary condition of Weierstrass in the multiple integral problem of the calculus of variations*, by Dr. E. J. McShane (National Research Fellow). (Abstract No. 37-3-143-*t*.)

14. *Note on a theorem of Bôcher and Koebe*, by Dr. J. J. Gergen. (Abstract No. 37-3-146-*t*.)

15. *Properties of the operator $z^{-v}\log z$, where $z=d/dx$*, by Professor H. T. Davis. (Abstract No. 37-3-148-*t*.)

16. *On an existence theorem of the calculus of variations*, by Professor L. M. Graves. (Abstract No. 37-3-151-*t*.)

17. *Analytic curves for which the chord equals the arc*, by Dr. J. M. Feld. (Abstract No. 37-3-152-*t*.)

18. *The generalized pedal transformations*, by Dr. J. M. Feld. (Abstract No. 37-3-153-*t*.)

19. *Birational contact transformations*, by Dr. J. M. Feld. (Abstract No. 37-3-154-*t*.)

20. *On the inverse of a certain Volterra functional transformation and its application to the evaluation of some definite integrals involving Bessel functions*, by Dr. H. P. Thielman. (Abtract No. 37-5-195-*t*.)

21. *Modular covariants*, by Professor Wilhelm Maier. (Abstract No. 37-5-196-*t*.)

GENERAL SESSION, Saturday morning.

22. *On the representative space S_9 of the plane cubics*, by Dr. Jacob Yerushalmy (National Research Fellow). (Abstract No. 37-3-147.)

23. *Conjugate nets and the lines of curvature*, by Professor E. P. Lane. (Abstract No. 37-3-144.)

24. *On the ruled $V_4^{(4)}$ in S_5 associated with a Schläfli hexad of 3-flats*, by Professor John Eiesland. (Abstract No. 37-3-150.)

25. *Homology groups associated with infinite complexes*, by Professor J. W. Alexander and Dr. L. W. Cohen (National Research Fellow). (Abstract No. 37-5-198.)

26. *A method for determining the canonical basis of an ideal in an algebraic field*, by Professor C. C. MacDuffee. (Abstract No. 37-3-142.)

27. *On certain divisors of the class number of a cyclotomic field*, by Professor C. G. Latimer. (Abstract No. 37-5-191.)

28. *On necessary and sufficient conditions for solvability of $f=0$, where f is any indefinite ternary quadratic form*, by Mr. A. E. Ross. (Abstract No. 37-5-200.)

29. *A class of maximum numbers and a class of minimum numbers that are associated with certain Diophantine equations of the form $\Sigma(1/x_1 x_2 \ldots x_r)=b/a$*, by Professor H. A. Simmons. (Abstract No. 37-5-190.)

30. *Improvements of the Cauchy lemma on simultaneous representation*, by Dr. Gordon Pall (National Research Fellow). (Abstract No. 37-5-194-*t*.)

31. *On sums of squares*, by Dr. Gordon Pall (National Research Fellow). (Abstract No. 37-5-193-*t*.)

32. *Expansion theory associated with linear differential equations and their regular singular points*, by Dr. Leonard Bristow. (Abstract No. 37-5-204-*t*.)

33. *The holoedric isomorphism of the collineation group G_{20160}, $P.G[2,2^2]$, and the ternary linear fractional group G_{20160} in G.F. $[2^2]$*, by Miss I. M. Schottenfels. (Abstract No. 37-5-210.)

34. *The behavior of the Hessian of a curve at a singular multiple point of the curve*, by Reverend J. E. Case. (Abstract No. 37-5-208-*t*.)

35. *On the analytical extension of the functions defined by factorial series*, by Professor H. K. Hughes. (Abstract No. 37-5-207-*t*.)

36. *Note on spaces satisfying the first enumerability axiom of Hausdorff*, by Mr. Selby Robinson. (Abstract No. 37-5-209-*t*.)

M. H. INGRAHAM,
Associate Secretary

THE APRIL MEETING IN BERKELEY

The two hundred ninetieth regular meeting of the Society was held at the University of California, Berkeley, California, on Saturday, April 11, 1931. The first session was called to order by Professor Uspensky at 10:30 a.m. The meeting was presided over later by Professor Hedrick.

There were present during the day some thirty-five persons, including the following twenty-three members of the Society:

N. M. Alderton, A. D. B. Andrews, H. M. Bacon, B. A. Bernstein, Thomas Buck, C. C. Craig, E. R. Hedrick, Harold Hotelling, D. H. Lehmer, D. N. Lehmer, S. H. Levy, E. W. McDonald, F. R. Morris C. A. Noble, T. M. Putnam, Saul Pollock, J. A. G. Shirk, Pauline Sperry, H. W. Stager, W. M. Whyburn, A. R. Williams, B. C. Wong, Kamcheung Woo.

The titles of papers read at the meeting follow. Those whose abstract numbers are followed by the letter *t* were read by title.

1. *On unit-zero Boolean representations of operations and relations*, by Professor B. A. Bernstein. (Abstract No. 37-3-155.)

2. *Note on the condition that a Boolean equation have a unique solution*, by Professor B. A. Bernstein. (Abstract No. 37-3-156-*t*.)

3. *Quadratic fields over the field of rational functions, modulo 2*, by Dr. Leonard Carlitz (National Research Fellow). (Abstract No. 37-3-159-*t*.)

4. *New Diophantine automorphisms*, by Dr. Leonard Carlitz (National Research Fellow). (Abstract No. 37-3-160-*t*.)

5. *A class of algebraic fields of characteristic p*, by Dr. Leonard Carlitz (National Research Fellow). (Abstract No. 37-3-158-*t*.)

6. *A problem in additive arithmetic*, by Dr. Leonard Carlitz (National Research Fellow). (Abstract No. 37-3-157-*t*.)

7. *On the composition of dependent elementary errors*, by Dr. C. C. Craig (National Research Fellow). (Abstract No. 37-5-211.)

8. *Note on the distribution of means of samples of N drawn from a type A population*, by Dr. C. C. Craig (National Research Fellow). (Abstract No. 37-5-212-*t*.)

9. *Sampling in the case of correlated observations*, by Dr. C. C. Craig (National Research Fellow). (Abstract No. 37-5-213-*t*.)

10. *On a property of the semi-invariants by Thiele*, by Dr. C. C. Craig (National Research Fellow). (Abstract No. 37–5–214–*t*.)

11. *Determinants and the real roots of an equation*, by Professor Raymond Garver. (Abstract No. 37–5–215–*t*.)

12. *The generalization of Student's ratio*, by Professor Harold Hotelling. (Abstract No. 37–5–216.)

13. *Arithmetic periodicity of Bessel functions*, by Dr. D. H. Lehmer (National Research Fellow). (Abstract No. 37–3–170.)

14. *On the arithmetic of double series*, by Dr. D. H. Lehmer (National Research Fellow.) (Abstract No. 37–3–171–*t*.)

15. *On factoring large numbers*, by Dr. D. H. Lehmer (National Research Fellow) and Mr. R. E. Powers. (Abstract No. 37–5–217–*t*.)

16. *Concerning certain one-parameter continuous functional groups and their integro-differential invariants*, by Professor A. D. Michal. (Abstract No. 37–5–218–*t*.)

17. *A theory of integral invariants in composite functional spaces*, by Professor A. D. Michal. (Abstract No. 37–5–219–*t*.)

18. *On the determination of line loci in four-space by a graphic method in geometry*, by Mr. Saul Pollock. (Abstract No. 37–5–220.)

19. *A property related to completeness*, by Professor J. H. Roberts. (Abstract No. 37–3–169–*t*.)

20. *Solutions of homogeneous linear difference equations by means of infinite determinants*, by Professor A. A. Shaw. (Abstract No. 37–5–221–*t*.)

21. *A problem in the geometry of numbers*, by Professor J. V. Uspensky. (Abstract No. 375–5–222.)

22. *System of Appell polynomials*, by Professor Morgan Ward. (Abstract No. 37–3–161–*t*.)

23. *Conditions for the solubility of the Diophantine equation*, by Professor Morgan Ward. (Abstract No. 37–3–163–*t*.)

24. *The linear form of numbers represented by a homogeneous*

polynomial in any number of variables, by Professor Morgan Ward. (Abstract No. 37–3–166–*t*.)

25. *Orthogonal Appell polynomials*, by Professor Morgan Ward. (Abstract No. 37–3–165–*t*.)

26. *Orthogonal and periodic systems of Appell polynomials*, by Professor Morgan Ward. (Abstract No. 37–3–164–*t*.)

27. *Some arithmetical properties of sequences satisfying a linear recursion relation*, by Professor Morgan Ward. (Abstract No. 37–3–162–*t*.)

28. *On the Lebesgue integral*, by Professor W. M. Whyburn. (Abstract No. 37–3–167.)

29. *Generalization of a theorem due to C. M. Cleveland*, by Dr. Leo Zippin (National Research Fellow). (Abstract No. 37–3–168–*t*.)

T. M. PUTNAM,
Associate Secretary

EVANS ON MATHEMATICAL ECONOMICS

Mathematical Introduction to Economics. By Griffith C. Evans. New York, McGraw-Hill Book Co., 1930. xi+177 pp. $3.00.

As the author remarks, this book is not a voluminous or complete treatise on mathematical economics, but is a short unified account of a sequence of economic problems by means of a few rather simple mathematical methods. Undoubtedly both the mathematician who knows little or no class-room economics and the economist who has only a knowledge of the integral and differential calculus can read this book with considerable profit.

In Chapters 14 and 15, Professor Evans gives a very short account of his own economic researches and those of other recent writers on dynamical economics. Professor Evans treatment of Divisia's instantaneous price index in Chapter 9, and his application of it to economic crises, is new and is not to be found elsewhere. His treatment of units and dimension theory in Chapter 2 is unusually good. Mathematicians will likely find these chapters most stimulating but all chapters are sufficiently mathematical to hold the interest of nearly any mathematician. The text is well suited for a course to follow the traditional course in differential and integral calculus. Such mathematical concepts as maxima and minima of functions and of integrals, elementary ordinary, partial and Pfaffian differential equations, and integral equations are employed in a way which a beginner can grasp. The teacher will, however, occasionally find himself called upon for more or less detailed explanations which the author omits. For example, without previous mention of tensors the author gives the following footnote on page 20: "Wealth may perhaps be regarded as a tensor or complex number, with one component for each kind of wealth; but the concept has not a great deal of significance, since it lacks application."

Instead of following the classical arrangement of material, that is, value, utility, marginal utility, etc., and losing the reader at the outset, Professor Evans prefers to begin with more or less popular conceptions of cost of production and demand and to postpone an involved study of demand by the use of utility functions to Chapters 11 and 12. The theory of production is considered in Chapter 13. The reader should supplement this treatment of production by the recent papers of Henry Schultz.

It is in Chapter 10 that Professor Evans explains his point of view with reference to the place of economic theory. He says, "Let us admit that the entire economic aspect of human affairs is necessarily too vast to be covered by a single theory. Our endeavor then should be to make systematic study of several groups of economic situations, as theoretical investigations, and bring out the respective hypotheses which separate these groups." The "bases of action" of individuals are various. "Sometimes there is an attempt to unify them by saying that a man tries to act in such a way as to increase his pleasure. But from this point of view we have to consider at the same time not only both capitalists and laborer, but also the profiteer and the soldier, the adventurer and the hermit, the teacher, the beggar and the thief."

The first chapter is, as labelled, an elementary theory of monopoly. Other subjects treated are monopoly, change of units, competition and cooperation of producers for the cases of fixed and variable prices, diversification of cost, tariff, rent, rates of exchange, theory of interest, and the equation of exchange.

In the discussion of competition of n producers, Professor Evans treats only the case for which all producers in the same market sell their products at the same price. We know from experience that this seldom happens. It is the reviewer's opinion that it is just about impossible to give a satisfactory treatment of the static problem of competition. One could write the n prices as functions of the n quantities produced. A theory of competition could then be built up for the case for which the operations of the producers were independent of each other, that is, the Jacobian not equal to zero, and for the case for which certain groups worked together, that is, the case for which the Jacobian vanishes. The large number of such price relations would of course materially complicate the problem. Professor Harold Hotelling has given a mathematical treatment to an important aspect of static equilibrium. He points out that there are groups of buyers associated with each seller who will deal with him in preference to others in spite of a moderate price difference which may vary continuously among buyers. In the case of the recent calculus of variations treatment of the dynamic problem of competition the difficulty concerning a single price seems to be obviated. Here it is proposed that customers compare the price of any competitor with an average price for the market, $p(t)$, where t represents time, and if the difference is sufficiently small make the purchase without further consideration. If all prices in the same market should remain constant, it might be reasonable to suppose that after a time all customers would go to the producer who maintained the lowest price (service, etc., of course, included); but when prices change as we know they do, the producer who has the high price today may have the low price of tomorrow. In other words prices are continually changing and the prices of all successful producers of an identical commodity in the same market fall within statistical limits of an ideal theoretical price function of the time.

It would seem that the author could have begun with an introduction of the concept of profit over an interval of time as represented by an integral. His mathematical tool would then have been the calculus of variations. The Euler-Lagrange equations of the calculus of variations would have been his own conditions for maximum and minimum without alteration, and by this single stroke he could have lifted the first six chapters from the unreal world of static economics to the promised land of economic dynamics. His omission is pardonable since this procedure might have had the effect of frightening many economists and others whose mathematical knowledge ends abruptly with the calculus, but the book seems hardly complete without a concluding chapter which indicates how this transformation could be easily accomplished.

Most of the misprints noted by the reviewer will be readily recognized by the reader. The following correction, however, is necessary to avoid confusion: page 12, last sentence of first paragraph of Exercise 11 should read: "show that if an advantage of profit is possible with a given expense of advertising z, it will be increased disproportionately by increasing that expense."

C. F. Roos

NEW EDITION OF OSGOOD ON FUNCTIONS

Lehrbuch der Funktionentheorie. By W. F. Osgood. Vol. I, fifth edition, 1928. 14+818. Vol. II, part 1, second edition, 1929. 7+307 pp. Leipzig and Berlin, Teubner.

The second edition of the first volume of this work was reviewed by E. B. Van Vleck in this Bulletin, (vol. 20 (1913), pp. 532–546). No reviews of the intervening editions have appeared in this journal. The first edition of the first half of the second volume was also reviewed by Professor Van Vleck, in this Bulletin (vol. 33 (1927), pp. 358–365). Both of these reviews are adequate and authoritative in giving an excellent idea of the plan of the work and of its chief characteristics. As a consequence we can limit ourselves in this review to the changes which have been made in these editions. In point of size, the fifth edition of the first volume contains some fifty odd pages more than the second. The added material is scattered through the volume, including changes in expressions, added exercises, added explanations, and new material.

The added material in the earlier chapters of the book include such matters as a separate section devoted to the definite (Cauchy) integral, its definition and properties, paragraphs on Abel's theorem on the continuity of a power series at a point of convergence on the circle of convergence, on iterated integrals, on the convergence of products of power series (it is puzzling why this was omitted in the early editions, when division and other arithmetic operations were included) and a section devoted to the Poincaré theta series. In the chapter on the logarithmic potential function the section on the motion of an incompressible fluid has been considerably expanded. The section covering the matter of isolated singularities of harmonic functions and the symmetry of Green's functions has been rearranged to advantage. Further, a paragraph giving the Arzelà theorem on compactness of a sequence of continuous functions has been added. The last chapter dealing with the uniformisation problem, aside from a number of minor changes, has been augmented by a section deriving, in detail, results concerning the analysis situs of the Riemann surfaces involved in the proof. Further, the final paragraphs, dealing with the uniformisation of an arbitrary analytic function, and the proof of the existence theorem for a many-valued function on an arbitrary region of definition, have been considerably revised. This has helped to make the subject matter more intelligible, the developments clearer, and have added materially to the usefulness of the volume, both as an introduction to the theory of functions, particularly the uniformisation problem, and as a work of reference.

Most of the exercises added to the later editions are of the thought provoking variety, frequently giving points of theory not covered in the text. One finds, for instance, as exercises, to prove that the sum of the residues of a rational function is zero, to discuss the many-valuedness of z^{α}, $(\alpha=a+ib)$, to prove that a monogenic analytic function can be at most denumerably infinitely many-valued, and so on. Putting questions of this character into problem form is certainly desirable, and could be carried to even further extremes.

If one were in a quibbling mood, one might take objection to the introduction of the definite integral on the basis of the most primitive form, of equal

subdivisions of the fundamental interval, as given in first courses in calculus. To be sure, it is adequate for the class of continuous functions, to which integration processes are restricted in this work. But certainly a mind which can cope with the intricacies of a proof of the Jordan curve theorem, and finds a proof of the Green's integration theorem based thereupon desirable, should be able to understand at least the Riemannian definition of integration.

Perhaps too, we might have expected some of the sets of references to have been brought up to date, for example, those on the Jordan curve theorem. On the other hand, it is a little disconcerting, when infinite derivates of real-valued continuous functions are admitted, to find a reference (p. 22) to a recent nowhere differentiable function, non-differentiability being restricted to nonexistence of a finite derivative, when E. H. Moore in 1900* called attention to a function which in a few lines can be shown to be non-differentiable in the sense of having at no point even an infinite derivative.

Another puzzling thing is that the author does not utilize the notion of total differential, as given on page 240, to advantage. It simplifies statements of differentiation properties of functions of two variables and gives a more elegant form to the condition for the existence of a derivative of a function of the complex variable. These more or less minor matters do not alter the fact that we have in this volume an example of fine and rigorous exposition.

The second edition of the first part of the second volume seems to have grown from 242 pages to 307 pages. Closer examination, however, reveals the fact that the type is larger, and the spacing wider, giving a more open and attractive page. The improvement in appearance is very commendable.

The contents of the volume have undergone only very slight changes. The first chapter is almost unchanged. The second chapter has received additional explanatory material, in a few places. Two sections, 14 and 15, dealing with the zeros of a single-valued function on a manifold, and the singularities of a pseudo-algebraic manifold, have been rewritten, considerably enlarged and improved. In the third chapter, the changes have been made in connection with the extension of the Weierstrass theorem that an everywhere meromorphic function is a rational function, to manifolds extended so as to include infinite regions, to which is then added the proof of the theorem that the most general one-to-one transformation of extended space into itself is a rational one, giving an elegant and satisfying conclusion to this part of the theory of functions.

Apropos of Professor Van Vleck's remark on this work in English, the following incident may be of interest. At a midwestern University, in a list of new books on mathematics received in the library of the university, occurred Osgood's *Theory of Functions*, translated into English. Investigation disclosed that the office secretary had by mistake sent out as received a list of books *wanted* by students.

T. H. HILDEBRANDT

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* Transactions of this Society, vol. 1 (1900), pp. 77 ff. The same example was rediscovered independently by Hahn: Jahresbericht der Vereinigung, vol. 26 (1918), pp. 281–284.

SHORTER NOTICES

Cours de Mathématiques Générales (Analyse et Géométrie). By R. Garnier. Tome I: Calcul Différentiel. Géométrie. Paris, Gauthier-Villars, 1930. 11+463 pp.

The French courses in *mathématiques générales* and *mathématiques spéciales* are designed to fill in the gap between the *Lycée* and the university. The former, offered in the university, is intended mainly for students of physics and chemistry who wish a practical technical knowledge of mathematics through advanced calculus. The emphasis is more on analysis than geometry.

The first eight chapters of the book under review are devoted to a classical exposition of differential calculus. The theory of infinite series, including calculations with expansions limited to a few terms, is given a much larger place in the exposition than is usual in American texts. The exponential function is introduced as a solution of the functional equation $f(x+y)=f(x)f(y)$. Again, the power series development of $y=(1+x)^{\alpha}$ is obtained as a particular solution of the differential equation $(1+x)y'=\alpha y$. Also the magnitude of the error committed in using Newton's method for computing the roots of an equation is discussed with unusual completeness.

The last three chapters deal with analytic geometry and elementary differential geometry. Vector analysis is used wherever feasible. The volume ends with the theorem of Meusnier and a discussion of the indicatrix of Dupin.

The reviewer noted only five slight typographical errors, on pages 181, 182, 221, 346, 447, which, however, do not rob the context of its significance. The style is clear. Some of the more abstract theorems are not proved, but references are made explicitly to Goursat, *Cours d'Analyse.* The large number of figures (172) should prove helpful to the student. The book is well written; possibly a good index, such as may be found in Vessiot et Montel, *Cours de Mathématiques Générales*, would be an improvement.

W. E. Byrne

Leçons de Géométrie Projective. By F. Enriques. Translated from the fourth Italian edition by P. Laberenne. Paris, Gauthier-Villars, 1930. 430 pp.

The German translation of this well known book was reviewed in this Bulletin by Snyder (vol. 10, 1904, p. 355), who called attention to the omission of any treatment of the ruled quadric surface. The French translation under review has an added Chapter XIV on ruled quadric surfaces and twisted cubic curves; and the former Chapter XIV has now become Chapter XV, with two added paragraphs on projective twisted cubics and elliptic biaxial homographies. Except for these additions, the book is a close translation of the earlier Italian and the German editions. Whether the added Chapter XIV was contained in the later Italian editions or appears for the first time in this French translation the reviewer is unable to say, as he has no convenient access to a copy of the fourth Italian edition.

W. B. Carver

A Short History of Mathematics. By Vera Sanford. Boston, Houghton Mifflin Company, 1930. xii+402 pp. $3.25.

The author of this scholarly volume has been eminently successful in focusing attention upon the development of the elementary branches of mathematics and their adaptations to the varied practical needs of the times.

There are abundant and well chosen illustrations; the material has been carefully selected; and the style is simple, clear, varied, and lively. The chapters on commercial mathematics and the place of mathematics in the school curriculum are perhaps the most unusual.

The book will be valuable not only for teacher training classes but also for certain teachers in service who have regarded high school mathematics as fixed in quantity and quality and have therefore been indiscriminately antagonistic toward all suggestions for modifying either the content or the methods in algebra or geometry.

This book is under the editorship of John Wesley Young and has an introduction by David Eugene Smith. For a more detailed review the reader is referred to The American Mathematical Monthly (vol. 37, 1930, pp. 540–542.)

ELIZABETH B. COWLEY

Die Differential- und Integralgleichungen der Mechanik und Physik. Edited by P. Frank and R. vonMises. Erster, Mathematischer, Teil, edited by R. von Mises, with the cooperation of L. Bieberbach, C. Carathéodory, R. Courant, P. Frank, R. Iglisch, K. Loewner, H. Rademacher, E. Rothe, G. Schulz, and G. Szegö. Zweite, vermehrte, Auflage. Braunschweig, Vieweg, 1930. xxiii+916 pp.

The first edition of this volume appeared in 1925 and was reviewed in this Bulletin, volume 33, page 365. The second edition is also labelled the eighth edition of Riemann-Weber's *Partiellen Differentialgleichungen der mathematischen Physik.*

The present volume is essentially of the same form as the preceding one but it has been worked over carefully to eliminate errors and to make additions which have increased the size from 687 to 916 pages.

The list of collaborators includes three names which did not appear in the earlier edition. Philipp Frank, the general editor of the second, or physical, part of the work, has contributed to Chapter 2 an article on the theory of groups of linear transformations and G. Schulz has contributed to Chapter 12 a section on singular integral equations. R. Iglisch has entirely rewritten Chapter 15 on boundary value problems in general and has added considerably to Chapter 18 on boundary value problems of partial differential equations of the second order.

Besides the additions made by the new contributors, Chapter 3, by K. Loewner, contains a more complete treatment of elliptic integrals and functions. In Chapter 17, G. Szegö has added to his discussion of the potential an article on the problem of the condenser and in Chapter 19, H. Rademacher and E. Rothe have extended their exposition of the special problems of partial differential equations connected with mathematical physics.

W. R. LONGLEY

NOTES

The American Association for the Advancement of Science will hold its first Summer Meeting at the California Institute of Technology, in Pasadena, on June 15–20, 1931. Section A (Mathematics) of the Association will meet on Tuesday, June 16; at that time the following addresses will be delivered by invitation of the program committee of the Section: *On certain Diophantine problems of approximation*, by Professor D. R. Curtiss; *Topics in potential theory*, by Professor O. D. Kellogg; *On integration in finite form*, by Professor J. V. Uspensky. The complete program of the entire meeting will appear in Science early in May.

The centenary of the birth of James Clerk Maxwell is to be celebrated at the University of Cambridge on October 1–2, 1931, following the Faraday celebration and the centenary meeting of the British Association for the Advancement of Science in London. Addresses will be given at Cambridge by Professors Einstein, Langevin, Larmor, Planck, Sir James Jeans, and Sir J. J. Thomson.

Cambridge University has awarded Smith's prizes to Mr. H. S. M. Coxeter, of Trinity College, and Mr. H. R. Hulme, of Gonville and Caius College, and Rayleigh prizes to Mr. H. Davenport, of Trinity College, Mr. B. Kuttner, of Christ's College, and Mr. J. C. P. Miller, of Trinity College.

The Royal Society of Arts offers a prize from the Thomas Gray Memorial Trust for the Improvement and Encouragement of Navigation for an essay on the stability of ships. Competition closes December 31, 1931.

On the occasion of its 150th anniversary celebration on March 17, 1931, the Manchester Literary and Philosophical Society presented its Dalton Medal to Sir J. J. Thomson.

The Echegaray medal of the Royal Academy of Sciences of Madrid has been awarded to Lord Rutherford.

The Franklin Institute, Philadelphia, has awarded a Franklin medal to Sir James Jeans, "in recognition of his many fruitful contributions to mathematical physics, especially in the realms of the dynamical theory of gases and the theory of radiation, and of his challenging explanations of astronomical problems and his illuminating expositions of modern scientific ideas."

The Astronomical Society of the Pacific has awarded its Bruce gold medal "for distinguished services to astronomy" to Dr. Willem de Sitter, director of the Observatory at Leiden.

Professor Elie Cartan has been elected a member of the Paris Academy of Sciences, as successor to the late Paul Appell.

E. Fabry has been elected a correspondent of the Paris Academy of Sciences in the section of geometry.

The following, among others, have been elected fellows of the Royal Society of Edinburgh: Dr. T. P. Black, head of the mathematics department of Trinity Academy, Leith; N. M. H. Lightfoot, lecturer in mathematics at Heriot-Watt College, Edinburgh; Dr. W. H. McCrea, lecturer in mathematics at the University of Edinburgh; Mr. H. S. Ruse, lecturer in mathematics, University of Edinburgh; Dr. J. Wishart, statistician, Rothampstead Experimental Station.

Sir William Bragg has been elected an honorary member of the British Institution of Electrical Engineers.

Professor E. C. Titchmarsh, of the University of Liverpool, has been recommended for election to the Royal Society of London.

Professor George David Birkhoff, of Harvard University, will give a series of lectures on *Le dernier théoreme de géometrie de Poincaré, ses généralisations, et ses applications à la dynamique*, at the Collège de France, during the last two weeks of April, under the Fondation Michonis.

A bust of Carl Friedrich Gauss, the distinguished mathematician, has been placed in the Massachusetts Institute of Technology, in memory of his great-grandson, Carl Friedrich Gauss, an alumnus of the Institute in the class of 1900. The bust, originally sent by the German government to the Chicago World's Fair in 1893, was recently presented to the Institute by the mother, sister, and brother of the late Mr. Gauss.

Professor E. B. Stouffer has been elected a member of the section committee of Section A of the American Association for the Advancement of Science.

Among those recently awarded Guggenheim fellowships are the following: Associate Professor E. D. Eastman, of the University of California, for studies in non-isothermal systems of ionic entropies and of electrons in metals; Professor Genaro Moreno Garcia-Conde, of the School of Military Engineering of Chile, for mathematical research, especially in the theory of functions of real variables; Professor J. R. Nielsen, of the University of Oklahoma, for theoretical and experimental studies of Raman spectra and molecular structure; Augusto Novaro, musicologist of Mexico City, for studies in the mathematical and physical theory of music and musical composition, and for research in the design and construction of new types of musical instruments; Associate Professor T. E. Phipps, of the University of Illinois, for studies of problems in the field of molecular rays; Associate Professor George Scatchard, of the Massachusetts Institute of Technology, for studies in the theory of liquid solutions.

Professor G. Doetsch, of the Stuttgart Technical School, has been appointed professor of mathematics at the University of Freiburg.

Dr. H. Jeffreys, of St. John's College, Cambridge, has been appointed to a readership in the faculty of mathematics of the University of Cambridge.

Dr. J. Wishart has been appointed university lecturer in statistics at the University of Cambridge.

Mr. G. U. Yule, of St. John's College, Cambridge, has been appointed reader in statistics.

The following doctorates in mathematics were conferred by American universities during 1930, in addition to those listed in the March number of this Bulletin:

C. M. Cleveland, Texas, June, minor in physics, *On the existence of acyclic continuous curves satisfying certain conditions with respect to a given continuous curve.*

J. B. Coleman, California, May, *Concerning the irreducibility of the characteristic equation of a ternary continued fraction.*

J. L. Dorroh, Texas, June, minor in physics, *Some metrical properties of descriptive planes.*

R. N. Haskell, Rice, June, minor in mathematical physics, *The mixed boundary value problem for Laplace's equation, in the plane.*

Professor Richard Courant, of Göttingen, will lecture at the University of California during the summer of 1932.

Professor R. H. Fowler, of Trinity College, Cambridge, will give a series of lectures on *Some recent developments in theoretical physics*, at the University of Wisconsin during April and May, 1931.

Dr. W. Heitler, of Göttingen, will be in residence at Ohio State University as visiting professor of theoretical physics.

Dr. A. A. Albert, of Columbia University, has been appointed assistant professor of mathematics at the University of Chicago.

Dr. Albert Eldred Currier and Dr. John Jay Gergen have been appointed Benjamin Peirce Instructors at Harvard University for the coming academic year.

Associate Professor George W. Hess, of Howard College, has been promoted to a professorship of mathematics.

Associate Professor Harold Hotelling, of Stanford University, has been appointed professor of economics at Columbia University.

Associate Professor R. B. Lindsay, of Brown University, has been appointed visiting professor of theoretical physics at the Polytechnic Institute of Brooklyn for the second semester of 1931–32.

Professor David Vernon Widder, of Bryn Mawr College, has been appointed assistant professor of mathematics at Harvard University.

The following appointments to instructorships in mathematics are announced:

Columbia University, Mr. E. R. Lorch;

Harvard University, Douglas Payne Adams, Allen Emil Anderson, James Sutherland Frame, Alan Stuart Galbraith, Lester Turner Moston, Sumner Byron Myers, Griffith Baley Price, Frederick Henry Steen, George Booth Van Schaack.

Mr. J. D. H. Dickson, senior fellow of Peterhouse, and author of numerous papers on thermodynamics and thermoelectricity, died February 6, 1931 at the age of eighty-one.

Mr. James Hammond, secretary to Sylvester during his professorship at Oxford, and known for his work in algebra, died August 29, 1930, at the age of eighty.

Dr. W. E. Johnson, lecturer in moral philosophy at Cambridge, and known for his work in logic, died January 14, 1931. at the age of seventy-two.

Mr. R. G. Lunnon, of Armstrong College, Newcastle, died January 25, 1931.

Professor Domenico Montesano, of the University of Naples, died October 1, 1930, at the age of sixty-seven.

Professor William Cain, of the University of North Carolina, is dead.

Dean H. L. Hodgkins, of George Washington University, is dead.

Dr. George Daniel Olds, President Emeritus of Amherst College and formerly professor of mathematics there, died on May 11, 1931, at the age of seventy-seven years. He had been a member of this Society since 1891.

Captain Henry Martin Paul, professor of mathematics in the United States Navy from 1897 to 1913, died March 15, 1931, at the age of eighty.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

172. Professor R. L. Jeffery: *Volterra integral equations of the first kind.*

This paper deals with a study of the equation (1) $f(x)=\int_0^x K(x,\xi)u(\xi)d\xi$ under the fgllowing conditions: (a) $f(0)=0$, $K(x,\xi)$ integrable in ξ for each x, $\{f(x)-f(x')\}/(x-x')$ and $\{K(x,\xi)-K(x',\xi)\}/(x-x')$ bounded; (b) the left hand derivatives $f'^{-}(x)$ and $K_{\bar{x}}(x,\xi)$ respectively of $f(x)$, and of $K(x,\xi)$ with respect to x, existing; (c) conditions on $\phi(x,x')=\{\int_{x'}^{x}K(x,\xi)d\xi\}/(x-x')$. If (a) holds and $\phi(x,x')$ is bounded uniformly from zero for x' sufficiently close to x, then functions $u_\epsilon(x)$ such that $|f(x)-\int_0^x K(x,\xi)u_\epsilon(\xi)d\xi|<\epsilon$ exist for any given ϵ. If (a) and (b) hold, and if $\lim_{x'\to x}\phi(x,x')=\rho(x)\not\equiv 0$, then (1) has a unique bounded solution given by a series involving $f'^{-}(x)$, $K_{\bar{x}}(x,\xi)$, and $\rho(x)$. If $\rho(x)\equiv 0$, let $\rho_1(x)$ be obtained by replacing $K(x,\xi)$ with $K_x(x,\xi)$ in the limiting process which gives $\rho(x)$. If then $\rho_1(x)\not\equiv 0$, $f''^{-}(x)$ and $K_{x\bar{x}}(x,\xi)$ both exist, there is no solution of (1) unless $f(0)=f'(0)=0$. If this last condition holds, equation (1) has a unique bounded solution given by a series involving $f''^{-}(x)$, $K_{x\bar{x}}(x,\xi)$, and $\rho_1(x)$. (Received March 12, 1931.)

173. Dr. Wladimir Seidel (National Research Fellow): *On analytic functions with infinitely many zeros.*

Let $f(z)$ be a regular, analytic function defined in the unit circle $|z|<1$. Consider a sequence of points $z_1, z_2, \cdots, z_n, \cdots$ lying in the interior of the unit circle and converging toward the point $z=1$. If $\lim_{n\to\infty}f(z_n)$ exists and is equal to α, we shall call α a *cluster value* of $f(z)$ in $z=1$, and we shall call the set of all cluster values of $f(z)$ in $z=1$ the *cluster set* of $f(z)$ in $z=1$. The author proves the following theorem: *Let $w=f(z)$ be a bounded analytic function: $|f(z)|<1$ in the unit circle $|z|<1$. Let $z_1, z_2, \cdots$ and $z_1', z_2', \cdots$ be two sequences of interior points of the unit circle converging toward $z=1$, such that the non-euclidean distance $D(z_n, z_n')$ is less than a constant M, independent of n: $D(z_n, z_n')<M$, $n=1, 2, \cdots$. Then $f(z)$ always has at least one of the following two properties*: (i) *If $\lim_{n\to\infty}f(z_n)=\alpha$, then $\lim_{n\to\infty}f(z_n')=\alpha$*; (ii) *The cluster set of $w=f(z)$ in $z=1$ contains a region R of the w-plane bounded by a closed Jordan curve and each value w, save at most one, of the region R is assumed infinitely often by $f(z)$ in the interior of $|z|<1$.* (Received March 13, 1931.)

174. Dr. L. S. Kennison: *Linear q-difference equations of continuous order.* Preliminary report.

The equation $\alpha(x)\phi(x)+\int_0^1 L(x,\theta)\phi(\theta x)d\theta=b(x)$ in the unknown function $\phi(x)$ is discussed, and a characteristic equation, in general transcendental, is obtained. Existence theorems for formal series solutions are obtained in this paper in certain cases, most of which apply to the corresponding homogeneous equation ($b(x)=0$). Application is made to Volterra integral equations. (Received March 12, 1931.)

175. Mr. S. B. Myers: *Adjoint systems in the problem of Mayer under general end conditions.*

Necessary conditions in the problems of Lagrange and Mayer with variable end points directly analogous to the Euler equations and transversality conditions in the ordinary calculus of variations problem in the plane have recently been given by Morse and the author (see the Proceedings of the American Academy of Arts and Sciences, vol. 66, No. 6, March 1931). The present paper shows that these necessary conditions can be derived from the criterion for the compatibility of the system adjoint to the differential and terminal equations of variation. The normal case is made a subcase of the abnormal. Such considerations refer a much larger part of the proofs back to the theory of differential equations—in particular, to the theory of adjoint systems—than is usually the case. (Received March 12, 1931.)

176. Mr. J. L. Doob: *The boundary values of an analytic function.*

For brevity, we say that $F(z)$, defined and measurable on $|z|=1$, has the metric cluster value α of the first (second) kind at e^{it} $(0\leqq t\leqq 2\pi)$ if at e^{it} the lower (upper) metric densities of the sets C_ϵ on which $|F(z)-\alpha|<\epsilon$ exceed, for all $\epsilon>0$, a constant $A>0$, independent of ϵ. We say, further, that $f(z)$, supposed analytic for $|z|<1$, has the cluster value α on the radius ρ_t to e^{it}, if $F(z)$ tends to α on some sequence of points on ρ_t tending to e^{it}. The chief results obtained here on cluster values are embodied in the following theorem. Suppose that $f(z)$ is bounded and analytic for $|z|<1$, and that $\lim_{r=1-0}f(re^{it})$, which then exists for almost all t on $(0, 2\pi)$, defines the function $F(e^{it})$. Then (i) if $F(z)$ has a cluster value α of the second kind at e^{it}, $f(z)$ has the cluster value α on ρ_t; (ii) if $F(z)$ has a cluster value α of the first kind at e^{it}, $\lim_{r=1-0}f(r_ne^{it_n})=\alpha$; (iii) if $\lim_{n\to\infty}(r_ne^{it_n})=1$, $(r_n<1)$, and $t_n^{p+1}/(1-r_n)$ and $\{1-\text{mes}(I_n\cdot C_\epsilon)/\text{mes}\ I_n\}/(\text{mes}\ I_n)^p$, $p>0$, where I_n is the arc $z=e^{it}$, $0\leqq t\leqq 2t_n$, are uniformly bounded for all n and $\epsilon>0$, then $\lim_{n\to\infty}f(r_ne^{it_n})=\alpha$. It is to be noted that the conditions in (ii) are satisfied almost everywhere on $|z|=1$, and that (ii) contains as a particular case the classical theorem of Pringsheim and others that $F(z)$ cannot have a discontinuity of the first kind (a jump). (Received March 13, 1931.)

177. Dr. Robin Robinson: *On the differential geometry of surfaces in non-euclidean space.*

In this paper an attempt is made to summarize and systematize the classical attack on differential geometry in non-euclidean space, particularly from the

point of view enunciated in W. C. Graustein's article *Invariant methods in classical differential geometry* (this Bulletin, vol. 36 (1930), p. 489). Secondly, a few new theorems on various types of surfaces are proved. Lastly, a discussion of surfaces of Gaussian curvature zero is made from a new point of view. Only for such surfaces does there exist a *characteristic function* whose directional derivatives along the lines of curvature are the geodesic curvatures of those lines. There are found necessary and sufficient conditions that a surface of Gaussian curvature zero exist having a given function as its characteristic function, and these surfaces are classified according to the nature of the functions defining them. (Received March 12, 1931.)

178. Dr. E. J. McShane (National Research Fellow): *On a certain inequality of Steiner.*

Steiner's inequality can be stated thus: If $f_1(x, y), f_2(x, y)$ are functions defined and continuous over the same portion of the xy plane, then the area of the surface $z=\frac{1}{2}(f_1(x, y)+f_2(x, y))$ is at most the arithmetic mean of the areas of $z=f_1$ and $z=f_2$. Conditions are obtained under which the equality is excluded, and applications are developed. The first application is to sequences of surfaces $z=z_j(x, y)$ tending uniformly to $z=z(x, y)$; it is shown that if the area of z_j tends to the area of z, $[(\partial z_j/\partial x-\partial z/\partial x)^2+(\partial z_j/\partial y-\partial z/\partial y)^2]^{1/2}$ converges in measure to zero. The second is to surfaces of least area: if a surface have for a given boundary curve the least possible area, any portion of it which projects orthogonally in a one-to-one way on some plane is analytic, and is a minimal surface in the sense of differential geometry. (Received March 14, 1931.)

179. Professor C. O. Oakley: *Semi-linear equations.* Part I: *Regular polygons.*

An equation of the form $u_0+m_1|u_1|+\cdots+m_n|u_n|=0$, where the m's are constants, and where $u_i=a_ix+b_iy+c_i$, $i=0, 1, \cdots, n$, we define as a semi-linear equation of order $p(\leqq n)$ if there are p and only p non-vanishing coefficients among the m's. The perimeter of any regular polygon, considered as a curve, is given by an equation of this form; and the point set made up of the perimeter of, and all points interior to, any regular polygon is also given by an equation of this same form. Special cases of certain theorems here stated were given without proof by Axel S¨derblom (*Sur l'Emploi de Valeurs Absolues dans la Géométrie Analytique*, G¨teborg, 1906, 174 pp.). (Received March 14, 1931.)

180. Dr. Jakob Levitzki: *On normal products of algebras.*

If the product AB of two algebras $A=a_1F+\cdots+a_sF$; $B=b_1F+\cdots+b_tF$ over the field F (where A and B are subsets of a given ring P) is a direct right-hand modul over B (that is, if every element of AB can be uniquely expressed as a right-hand linear form with coefficients in B) and if every element a_i is commutative with the algebra B (not necessarily with every element of B) then the product AB is called a normal product with respect to B and is an algebra over F. A necessary and sufficient condition is given that this algebra

be semi-simple. Further, the different representations of a given semi-simple algebra as a normal product of its subalgebras are studied and applications to group-algebras are made. (Received March 12, 1931.)

181. Dr. Oscar Zariski: *On the irregularity of cyclic multiple planes.*

Cyclic multiple planes $z^n=f(x, y)$ are considered, where n is an arbitrary positive integer, and $f(x, y)=0$ is an algebraic curve of order m, possessing nodes and cusps only. A formula for the irregularity $q=p_g=p_a$ of the surface is given in terms of the superabundances of certain linear systems of curves in the (x, y)-plane, determined by simple basis points at the cusps of the curve f. If $f=0$ is an irreducible curve, the following theorem holds: A necessary and sufficient condition that the surface be irregular is that m and n be divisible by 6, and that, putting $m=6j$, the linear system of curves of order $m-3-j$ passing through the cusps of the curve f should be superabundant. If these conditions are satisfied, the irregularity of the surface does not depend on n and is equal to the superabundance of the mentioned system of curves. An application to the theory of cuspidal curves is the following. The cusps of an irreducible plane algebraic curve of order m, possessing nodes and cusps only, impose independent conditions on the curves of any order $m-3-\alpha$ constrained to contain them, as long as α remains less than $m/6$. A noteworthy consequence of this theorem is the existence of sets of 6 non-negative integers which satisfy the Plücker relations and which are not the Plückerian characters of any effectively existent plane algebraic curve. Examples of such illusory Plückerian characters are given here for the first time. (Received March 12, 1931.)

182. Dr. Oscar Zariski: *On illusory Plückerian characters.*

In a paper which will appear in the Annals of Mathematics, the author proves that the well known question as to whether any preassigned set of Plückerian characters always corresponds to an effectively existent plane algebraic curve should be answered in the negative. For instance, there do not exist curves of order 7 with 11 cusps, or curves of order 8 with 16 (or more) cusps. The general theorems on which the proof is based form the subject of another communication (*On the irregularity of cyclic multiple planes*) and involve the elements of the theory of algebraic surfaces. Here a simple direct proof is given of the non-existence of self-dual curves of order 8 with 16 cusps, which makes use only of the elements of plane geometry. (Received March 12, 1931.)

183. Dr. R. P. Agnew (National Research Fellow): *On equivalence of methods of evaluation of sequences.*

We show that if the coefficients in a regular transformation (T): $T_n=\sum_{k=1}^{\infty}a_{nk}s_k$ satisfy certain conditions, then (T) is equivalent to convergence. As an application of these results, we find sufficient conditions for equivalence of (T) and the Cesàro arithmetic-mean transformation. (Received March 25, 1931.)

184. Professor R. Vaidyanathaswami: *The theory of multiplicative arithmetic functions.*

The author presents a theory of the class of functions $f(M_1, M_2, \cdots, M_r)$ of r positive integral arguments, which satisfy the relation $f(M_1N_1, M_2N_2, \cdots, M_rN_r) = f(M_1, M_2, \cdots, M_r)f(N_1, N_2, \cdots, N_r)$ whenever the products $M_1M_2 \cdots M_r$, $N_1N_2 \cdots N_r$ are relatively prime. (Received April 3, 1931.)

185. Professor Jesse Douglas: *The Koebe mapping theorem and the problem of Plateau.*

The mapping theorem of Koebe meant here is that a polyhedral surface can be mapped conformally on a circular disc. The present paper shows that, starting with this theorem, the proof of the existence of a minimal surface bounded by any Jordan contour is a matter of a few supplementary remarks of an evident nature. The paper will appear in the Abhandlungen aus dem Mathematischen Seminar der Hamburgischen Universität. (Received April 3, 1931.)

186. Professor J. F. Ritt: *Systems of algebraic differential equations.*

This paper describes a theoretical process for resolving a system of algebraic differential equations into irreducible systems. The process consists in differentiating the given system of equations a sufficient number of times and resolving the enlarged system into systems which are irreducible algebraically. A second result of the paper is that, given any system of algebraic differential equations, in unknown functions $y_1, \cdots, y_n$, there exists an equivalent system composed of $n+1$ or fewer equations. (Received March 24, 1931.)

187. Mr. H. L. Dorwart and Professor Oystein Ore: *Certain types of criteria for irreducibility of polynomials.*

Various authors, Schur, Pólya, Stäckel, etc., have devised irreducibility criteria for polynomials, based on the values which these polynomials can take. In this paper is shown how these theorems can be deduced and generalized by means of elementary theorems on the arithmetical properties of polynomials. (Received March 12, 1931.)

188. Professor Warren Weaver: *The Gaussian law of error.*

Most proofs of the Gaussian law of error fall under two types: those which assume that the most probable (or the expected) value following a set of observations is the arithmetic mean of the set; and those which assume the error to be the sum of a very large number of elementary errors. A critical resumé of such proofs is given, pointing out their limitations. Two new proofs of the Gaussian law are then given. The first assumes that, following n observations, the a posteriori probability function for the true value is symmetric with respect to the arithmetic mean of the observations. It is then proved that the error law, if not everywhere discontinuous, is the Gaussian law. The second proof assumes symmetry of the a posteriori probability with respect to some

point x_0 which is an unknown function of the observations. It is again proved that the error law, if not everywhere discontinuous, is Gaussian, and that x_0 is the arithmetic mean of the observations. (Received March 9, 1931.)

189. Professor J. D. Grant: *Second degree addition theorems for even functions.*

It is shown that an even solution of an addition theorem of second degree in $F(x+y)$ is a linear fractional transformation of the Weierstrass "P" function or one of its degenerate forms. (Received March 9, 1931.)

190. Professor H. A. Simmons: *A class of maximum numbers and a class of minimum numbers that are associated with certain Diophantine equations of the form* $\Sigma(1/x_1x_2 \cdots x_r)=b/a$.

For the Diophantine equation (1) $\Sigma(1/x_1x_2 \cdots x_r)=b/a$, $a=(c+1)b-1$, in which c, b are positive integers, the author recently exhibited a "principal" solution $x=w$ (American Mathematical Monthly, March, 1930). If (1) is satisfied by n numbers $x_i=e_i$, $(i=1, \cdots, n)$, where $e_1 \leqq e_2 \leqq \cdots \leqq e_n$, and if the first $n-1$ of these numbers are positive integers, the set e is called a solution ρ of (1). Let $P(x)$ stand for any polynomial that is symmetric in n variables $x_1, x_2, \cdots, x_n$ and has only positive coefficients. Theorem 1. If $x \neq w$ is a solution ρ of equation (1), $P(x)<P(w)$. The author obtains the following generalization of the known result concerning the maximum number that exists in a solution ρ of $\Sigma(1/x_i)=b/a$, a being as in (1): Theorem 2. The w_n of solution w is the largest number that exists in any solution ρ of equation (1) and w_n appears in no solution ρ of (1) except w. In connection with the set $x=W$, where $W_i=[(a/b)\binom{n}{r}]^{1/r}$, $i=1, \cdots, n$, the following is true: Theorem 3. If $x \neq W$ satisfies (1) and has $x_i \geqq 1$, $(i=1, \cdots, n)$, then $P(W)<P(x)$. (Received March 9, 1931.)

191. Professor C. G. Latimer: *On certain divisors of the class number of a cyclotomic field.*

Let F be an algebraic field of degree E and a primary divisor of the field defined by a primitive Mth root of unity. Assume that neither E nor M contains a square factor >1. We shall show that the number of narrowly equivalent ideals in F is divisible by $T=e_1^{k_1-1}e_2^{k_2-1}\ldots e_n^{k_n-1}$, where the e's are the prime divisors of E and each k_i is the number of elements $\neq 0$ in the ith row of a certain matrix (γ_{ij}). (γ_{ij}) is the matrix of the system of linear congruences which define the group to which F belongs. (Weber, *Lehrbuch der Algebra*, vol. 2, p. 100.) It is shown that the classes of ideals in F may be segregated in T genera, each containing the same number of classes. For the case $E=2$, this result is well known. (Received March 10, 1931.)

192. Professor A. E. Young: *Automatic integration and electrical computing instruments.*

Mathematical physicists have developed theories and established equations and formulas which serve as the foundation for various types of measurement of the utmost value in the industrial world. The formulas, almost without excep-

tion, are differential in form, the integrand being expressed as a function of one or more variables whose values, directly measurable, are varying with the time. To use these properly in industrial measurements requires a time determination and summation of the values of the integrand, or in other words, an integration with regard to the time. With a few important exceptions the integration procedure in the past has been replaced by computations based on reference tables and on daily chart records of the values of the time-variables. The author has invented automatic electrical instruments which sum with great frequency and accuracy various types of such integrand elements and thereby, for any particular time-interval, computes the sum whose limit is the time integral for the period. (Received March 10, 1931.)

193. Dr. Gordon Pall (National Research Fellow): *On sums of squares.*

Several results are given concerning sums of three positive squares; for example, every positive integer, except $25 \cdot 4^h$, which is a sum of three squares and possesses an odd square factor, is a sum of three positive squares. Application is made of an elegant formula for $r_s(cn)^2/r_s(c)$, where $r_s(t)$ is the number of representations of t as the sum of s squares, and $s=2, 3, 4, 5, 6, 7, 8, s=10$ and c not a sum of two squares, $s=11$ and c not a sum of three squares, or $s=12$ and c is even. All numbers not sums of four unequal squares are determined. (Received March 11, 1931.)

194. Dr. Gordon (Pall National Research Fellow): *Improvements of the Cauchy lemma on simultaneous representation.*

The condition $b^2+2b+4>3a$ of the Cauchy lemma on the simultaneous decomposition of an integer a as a sum of four integral squares and an integer b as the sum of the four bases taken $\geqq 0$, is here much improved whether a, b are even or odd. The method is elementary and uses the simplest rational automorphs of a sum of three squares. Some results of the type that certain forms of n possess representations as a sum of three squares with certain special properties are obtained incidentally. (Received March 11, 1931.)

195. Dr. H. P. Thielman: *On the inverse of a certain Volterra functional transformation and its application to the evaluation of some definite integrals involving Bessel functions.*

In this paper formulas are derived which express the integral of products of Bessel functions of any order between certain limits in finite terms of algebraic and trigonometric functions. These results are obtained by the use of a certain Volterra transformation and its inverse, the Laplace transformation. (Received March 11, 1931.)

196. Dr. Wilhelm Maier: *Modular covariants.*

Extending earlier results and methods (*Elementary properties of the t-functions*, Transactions of this Society, vol. 32, (1930)) the author introduces many-

valued analytic functions of two variables, which are closely related to Kronecker's limit formula, and develop their simplest properties. (Received March 11, 1931.)

197. Dr. Wilhelm Maier: *Integral equation of the* Γ*-function.*

Let $\Gamma(s)$ be the classical function of Euler; then there exists a quadratic integral equation, which is satisfied by $\Gamma(s)$. Moreover, that integral equation with initial conditions characterizes the analytic function $\Gamma(s)$ uniquely, and may be taken as the starting point of the Γ-theory. (Received March 11, 1931.)

198. Professor J. W. Alexander and Dr. L. W. Cohen (National Research Fellow): *Homology groups associated with infinite complexes.*

In developing the theory of homology for infinite complexes, there are two definitions of bounding which may be taken as points of departure. These definitions are distinct in the case of cycles with integral coefficients but are equivalent in case the coefficients are residues modulo an integer or are rational numbers. A natural definition of a distance function over the elements of the group suggests itself. The groups are shown to be separable, complete spaces. The modular groups are compact. The groups have denumerable bases. The question of a normal form for the groups is treated and settled in certain cases. (Received March 11, 1931.)

199. Dr. I. Schoenberg (International Research Fellow): *Note on geodesics in higher spaces.*

In recent years geodesics have been studied in the so-called non-holonomic spaces, that is, ordinary Riemann spaces whose curves are constrained to satisfy a given system of Pfaffian equations. There are two different kinds, the mechanical and the geometrical geodesics. The mechanical geodesics are suggested by the dynamical trajectories of mechanical non-holonomic systems and are not the solutions of an ordinary problem of the calculus of variations, while the geometric geodesics have the usual property of such curves in being the extremals of a problem of Lagrange. The author shows in the same way as for geodesics in Riemannian spaces that for these extremals the necessary conditions of Legendre and Weierstrass are always satisfied. To insure a minimum property of the geodesics joining two fixed points it is therefore sufficient to check up Jacobi's condition. This latter condition is also discussed. (Received March 12, 1931.)

200. Mr. A. E. Ross: *On necessary and sufficient conditions for solvability of* $f=0$*, where* f *is any indefinite ternary quadratic form.*

H. J. S. Smith (Proceedings of the Royal Society, vol. 13 (1864), pp. 110–111) stated conditions for solvability in integers not all zero of $f=0$, for a general indefinite ternary quadratic form f, but gave no proof. A. Meyer (Journal für Mathematik, vol. 98 (1885), pp. 177–179) proved Smith's assertion for the

case of forms with the odd-determinants. Employing a result of a former paper (this Bulletin, vol. 36, (1930), p. 56), there is given here a proof of the Legendre criterion for solvability of $ax^2+by^2+cz^2=0$ (Legendre, Mémoir, Académie des Sciences de Paris, 1785, pp. 507–514; also see L. E. Dickson, *Introduction to the Theory of Numbers*, p. 117). Using this latter, there is deduced, in a very simple manner, the theorem of Smith without restriction on the determinant of f. (Received March 12, 1931.)

201. Mr. A. T. Craig: *The frequency of x/y for correlated variables.*

Pearson and others have considered the frequency function of x/y and certain approximations have been found for both the correlated and independent cases. In the present paper, the exact frequency laws of x/y, xy, $x+y$, and $x-y$ are obtained when x and y are normally correlated variables. For the special case of x and y independent, the results are valid upon placing $r=0$. When the correlation surface of x and y is of a special non-normal form given by Professor H. L. Rietz in an unpublished paper, the laws of frequency of x/y, xy, $x+y$, and $x-y$ are also given. (Received March 12, 1931.)

202. Mr. Rufus Oldenburger: *Reduced matrices of class 3.*

Certain matrices of class 3 can be associated with general quadratic homogeneous and quadratic Cremona transformations. On the basis of file ranks of such matrices such transformations can be reduced to certain simple forms. The study of such matrices shows that any 2-way couche of a general cubic matrix (a_{ijk}) can under certain conditions be made symmetric under non-singular transformations and reduced to canonical form depending on the rank of this couche. The conditions under which a further couche can be reduced to canonical form without changing the couche already reduced are treated for cases in which (a_{ijk}) is of range 3 and 4, corresponding to plane and space transformations, and for a general range n where this couche is of any file rank. The geometric properties of certain reduced transformations are studied. It is shown that if $n=3$ a couche of rank one determined by fixing one of the symmetric indices can never be a matrix associated with a Cremona transformation. (Received March 12, 1931.)

203. Professor H. W. March: *The elastic instability of members with thin outstanding flanges of non-isotropic material.*

The methods used by Timoshenko in discussing problems of elastic instability for structural members of isotropic material are extended to members of non-isotropic material such as wood having three mutually perpendicular planes of elastic symmetry. A flange is treated as a rectangular elastic plate having the two opposite edges to which the compressive load is applied simply supported, one edge free, and the remaining edge either perfectly fixed, partially fixed or simply supported. This mathematical investigation forms part of a report on elastic instability prepared in collaboration with G. W. Trayer at the U. S. Forest Products Laboratory for the National Advisory Committee for Aeronautics. (Received March 12, 1931.)

204. Dr. Leonard Bristow: *Expansion theory associated with linear differential equations and their regular singular points.*

This differential equation is taken in the form $x^{n-1}y^{(n)}+x^{n-2}ay^{(n-1)}+x^{n-3}\rho_2(x)y^{(n-2)}+\cdots+\rho_n(x)y/x+\lambda y/x=0$, where $\rho_2(x), \cdots, \rho_n(x)$ are analytic functions of the complex variable x at $x=0$. The problem is set up as a boundary value problem. Suitable boundary conditions are determined together with characteristic values of λ. An expansion of $(t-x)^{-1}$ is obtained in a series of the solutions of the differential equation with argument x. The coefficients are polynomials in t^{-1} which are associated with a corresponding solution of the adjoint equation. By means of Cauchy's theorem there are obtained the expansions of (a) an arbitrary function analytic at the origin, (b) an arbitrary function analytic in an annular ring, (c) an arbitrary function of more than one complex variable analytic in suitable regions. The expansions converge absolutely and uniformly with respect to x in regions bounded by circles. (Received March 13, 1931.)

205. Professor H. S. Wall: *General theorems on the convergence of sequences of Padé approximants.*

Theorem I. Let $P(z)$ be a convergent series of Stieltjes. Then every infinite sequence of distinct approximants converges to $P(z)$ uniformly over K, where K is an arbitrary closed region lying entirely within the circles of convergence of $P(z)$ and of its reciprocal. Theorem 2. If $P(z)$ is a series of Stieltjes which is summable (Borel) to $F(z)$, then the diagonal files S_k, $k\geqq -1$, converge to $F(z)$. The same holds for $k<-1$ if the reciprocal of $P(z)$ is summable. Theorem 3. If $P(z)$ is a convergent positive definite series, then $S_k(k=0, \pm 1, \pm 2, \cdots)$ converges to a common limit $F(z)$. It is shown that the Stieltjes summability (as applied to positive definite series) is more inclusive than Borel summability. (Received March 14, 1931.)

206. Professor W. S. Kimball and Mr. W. J. King: *Theory of heat conduction and convection from a hot vertical plate.*

An analysis is made of free conduction and convection in air opposite a hot vertical plate, and checked by recent experiments of German investigators. Two new approximate empirical laws are employed to supplement the slightly modified classical theory. Law I: The locus of the maxima of the convection velocity curves is an isothermal surface: $x=(T_1-T_0)/[2a(1+be^{-ay})]$, at temperature half way between T_1, the hot plate temperature, and T_0, the ambient air temperature. Law II: Half the heat is convected up away inside this surface, or "film" boundary, and half outside. This theory is consistent with the five-fourths power law for variation of heat transfer, and with known facts concerning maximum convection velocities, heat transfer coefficients, and variations of film thickness. Two theorems concerning rate of heat transfer by convection and viscosity effects are given. (Received March 24, 1931.)

207. Professor H. K. Hughes: *On the analytical extension of functions defined by factorial series.*

This paper is concerned with the analytical extensions of functions such as are defined by series of the forms (1) $\sum_{n=0}^{\infty} g(n)/[z(z+1)(z+2)\cdots(z+n)]$ and

(2) $g(0)+\sum_{n=0}^{\infty}g(n)(z-1)\cdot(z-2)\cdots(z-n)$. It is well known that the region of convergence of a series such as (1) or (2) is in general a half-plane, lying to the right of a vertical line in the z complex plane. The author obtains expressions which, under certain restrictions on the general coefficient $g(n)$, furnish the analytical extension of the function defined by the given series (1) or (2) throughout the whole finite z-plane, the neighborhoods of the points $z=0$, -1, -2, $\cdots$ being excepted. (Received April 4, 1931.)

208. Reverend J. E. Case: *The behavior of the Hessian of a curve at a singular multiple point of the curve.*

So far as known, there is no literature on the general case as treated in this paper. Certainly the method of attack is new. The object of this paper is to find a method of placing the Newton polygon of the Hessian of an algebraic plane curve defined by $f(x, y)=0$ on the analytic triangle and by comparing it with the Newton polygon of the curve to discover the relationship between the curve and its Hessian at a multiple point of the curve. The method is based on a summation form of writing the Hessian; this is thought to be original. Within the limits of this summation all the points of the analytic triangle which contribute to the Newton polygon will be found. A knowledge of the terms of $f(x, y)$ which contribute to its Newton polygon is sufficient to determine the Newton polygon of the Hessian. There is a definite relationship between the branches of the Hessian and the branches of the original curve. The nature of the Hessian at a multiple point of its original curve is completely known when the degrees of the terms which contribute points to the Newton polygon of the curve are known. (Received April 4, 1931.)

209. Mr. Selby Robinson: *Note on spaces satisfying the first enumerability axiom of Hausdorff.*

In this paper it is proved that the first enumerability axiom and the first three properties of Riesz together constitute a necessary and sufficient condition that a space be a space V as defined by Fréchet in *Espaces Abstraits*. In a space V the following properties are shown to be equivalent: the fourth property of Riesz, property D of Hausdorff, and the property of being an L space. It is shown by an example that a space V need not be an L space. If a point has a well ordered decreasing family of neighborhoods equivalent to the family of all neighborhoods of the points, sets of conditions are given which are sufficient to insure that this decreasing family can be replaced by an enumerable sub-family. An example is given of a separable, compact, completely normal space which satisfies the first enumerability axiom but not the second. (Received April 4, 1931.)

210. Miss I. M. Schottenfels: *The holoedric isomorphism of the collineation group G_{20160}, $P.G[2, 2^2]$, and the ternary linear fractional group G_{20160} in $G.F.[2^2]$.*

U. G. Mitchell in his doctor's dissertation (Princeton, 1910) designates the group of all projective transformations in $P.G[2, 2^2]$ as G_{60480}; its self-conjugate sub-group of determinant unity is the simple group G_{20160}. The author proves

that this group is holoedrically isomorphic with the ternary linear fractional group in the Galois field $[2^2]$. (Received April 4, 1931.)

211. Dr. C. C. Craig (National Research Fellow): *On the composition of dependent elementary errors.*

If the probability variable x is the sum of n equal and independent component probability variables, $z_1, z_2, \cdots, z_n$, Cramér has shown that under certain quite general conditions the representations of the probability and frequency functions of x by a Charlier type A series (series in Hermite polynomials) actually has asymptotic properties with respect to n. (Skandinavisk Aktuarietidskrift, 1928, pp. 13–74). It is well known that under very wide conditions the probability function of x approaches the Gauss-Laplace law as n increases indefinitely. Bernstein has shown that this latter fact still holds when the condition of independence on $z_1, z_2, \cdots, z_n$ has to a considerable extent been removed (Mathematische Annalen, 1927, pp. 1–59). The present paper, by a combination of the methods of these two investigators, shows that the type A representation of the probability and frequency functions of x still retains its asymptotic property when the independence condition on $z_1, z_2, \cdots, z_n$ has to a certain extent been removed. (Received March 17, 1931.)

212. Dr. C. C. Craig (National Research Fellow): *Note on the distribution of means of samples of N drawn from a type A population.*

Recently Dr. George A. Baker has found the distribution of means of samples drawn at random from a population whose frequency function is a Gram-Charlier series of a finite number of terms. (Annals of Mathematical Statistics, vol. 1, pp. 199–204). It is the purpose of this note to call attention to the fact that by the use of the semi-invariant notation, Dr. Baker's results may be reached much more simply and directly. (Received March 17, 1931.)

213. Dr. C. C. Craig (National Research Fellow): *Sampling in the case of correlated observations.*

There are many important cases in which the usual assumption of independence or randomness in the set of observations which compose a sample of N is not justifiable. In the present paper, by the use of the semi-invariants of Thiele, a method is developed for the calculation of the characteristics of the distribution of moments about the mean in such samples, which is particularly adapted to the case in which the N observations are normally correlated. General formulas for computation in this case are given. These are applied to the particular case in which only consecutive observations are correlated and in a constant degree and results are given in addition to the mean and variance of the variance given by Rhodes (Journal of the Royal Statistical Society, 1927, pp. 135–143). A by product is a proof that if all the observations are normally correlated with each other and in equal degree, the distribution of all the moments about the mean in samples of N is the same in form as if the observations were independent. (Received March 17, 1931.)

214. Dr. C. C. Craig (National Research Fellow): *On a property of the semi-invariants of Thiele.*

Given a linear form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n$$

of a set of probability variables, $x_1, x_2, \cdots, x_n$, it is one of the most useful and important properties of the semi-invariants of Thiele that the rth semi-invariant of this form is, in the case where $x_1, x_2, \cdots, x_n$ are independent, simply

$$a_1{}^r\lambda_r{}^{(1)} + a_2{}^r\lambda_r{}^{(2)} + \cdots + a_n{}^r\lambda_r{}^{(1)},$$

in which $\lambda_r{}^{(i)}$ is the rth semi-invariant of x_i. This paper shows that among such isobaric functions of the moments of several variables this property is characteristic of the semi-invariants. Further, it is also shown that the necessary and sufficient condition that there be other such functions of weight $\geqq k$ in special cases is that the probability functions of each x_i, in case of independence, be such that, for each, λ_r vanishes for some $r < k$. (Received March 17, 1931.)

215. Professor Raymond Garver: *Determinants and the real roots of an equation.*

Borchardt and Weber have shown that the number of distinct roots, and of distinct real roots, of an equation with real coefficients can be stated in terms of the rank and the signs of the principal minors of certain symmetric determinants. Borchardt's determinant is the usual determinant form of the discriminant of the equation, while that used by Weber has lower order by one. In this paper, another such determinant is developed, with the aid of which the relation between the other two, which is not at all obvious, becomes apparent. (Received March 23, 1931.)

216. Professor Harold Hotelling: *The generalization of Student's ratio.*

The problem of simultaneous estimation of deviations in several variates has been attacked by Pearson and by Romanovsky with several "coefficients of racial likeness," but these are valid only on the assumption that the variates are independent, and they have excessively complicated sampling distributions. The need for simultaneous estimates extends outside of biometry, and includes the problem of comparing correlated variates such as prices at different times of a list of articles. The author recommends a new measure T of simultaneous deviations which, for a single variate, reduces to Student's ratio of mean to standard error. Putting a^{ij} for the covariance *estimated from the sample* of the statistics ξ^i and ξ^j (means, regression coefficients, etc.), a_{ij} for the tensor conjugate to a^{ij}, and n for the number of degrees of freedom among the columns of the reduced matrix of observations, we define the proposed statistic by

$$nT^2 = \sum\sum a_{ij}\xi^i\xi_0.$$

Its invariantive character shows that its distribution is independent of the covariances in the population, and this facilitates the derivation of this distribution, which is shown to be reducible to the incomplete beta function and hence to Fisher's distribution for the analysis of variance. (Received March 23, 1931.)

217. Dr. D. H. Lehmer (National Research Fellow) and Mr. R. E. Powers: *On factoring large numbers.*

Various non-tentative methods of factoring the odd number N based on the expansion of $N^{1/2}$ in a regular continued fraction have been described. Most of the methods that have been suggested depend for their success upon the appearance of a perfect square among the denominators of the complete quotients of even order. Two methods are now available when the product of any number of these denominators is a square, an event that occurs quite frequently in practice. The object of each method is to obtain a difference of squares divisible by N. To do this the first method uses the numerators of the complete quotients, and the second method the numerators of the convergents. These methods are described, compared and illustrated. Proof is also given of the fact that the success of one method implies the success of the other, so that one is free to use whichever method is easiest to apply. (Received March 18, 1931.)

218. Professor A. D. Michal: *Concerning certain one-parameter continuous functional groups and their integro-differential invariants.*

The first part of the paper is concerned with the functional invariants $f[y'(x), y(a), z]$ under the group composed of the one-parameter Volterra group in $y(x)$ and the translation group $z_1 = z + \tau$. Extensions to the case of integro-differential invariants of order n are also taken up. The second part of the paper deals with functional invariants of one-parameter functional groups arising from Cauchy problems of certain second-order partial differential equations of mathematical physics. (Received March 23, 1931.)

219. Professor A. D. Michal: *A theory of integral invariants in composite functional spaces.*

This paper is concerned with a theory of integral invariants in the composite functional spaces whose points have coordinates $(y_1^\alpha, y_2^\alpha, \cdots, y_n^\alpha, t)$. The paper will appear in an early issue of the Proceedings of the National Academy of Sciences. (Received March 23, 1931.)

220. Mr. Saul Pollock: *On the determination of line loci in four-space by a graphic method in geometry.*

In Wong's paper *On certain loci of lines incident with curves and surfaces in four-space* (this Bulletin, vol. 35 (1929), pp. 353–358) a list of 27 formulas for the orders of the ruled loci whose lines satisfy four, five and six simple conditions is derived by a synthetic method. It is the purpose of this paper to show how a new method (graphic method) may be employed to derive the formulas given by Wong and formulas for the special cases (where they exist) for each of the 27 formulas mentioned and additional formulas not included in the list. The graphic method is a method by decomposition applicable to general varieties in S_n. It was first introduced by Wong in a paper *On the number of apparent triple points of surfaces in space of four dimensions* (this Bulletin, vol. 35 (1929), pp. 339–343). (Received April 6, 1931.)

221. Professor A. A. Shaw: *Solutions of homogeneous linear difference equations by means of infinite determinants.*

It is the purpose of this paper to give solutions of homogeneous linear difference equations by use of infinite determinants. From a given homogeneous linear difference equation we derive an infinite system of equations with infinitely many unknowns. We solve this system by infinite determinants, using the "heuristic" method. The first part of the paper gives the solution of $a_x u_{x+2}+u_{x+1}+a_x u_x=0$, where a_x is any arbitrary function of x. The laws of expansions of the determinants are proved by mathematical induction and their convergence by use of von Koch's lemma. The second part of the paper deals with the solution of $b_x u_{x+2}+u_{x+1}+a_x u_x=0$, where a_x and b_x are arbitrary functions of x, treated in the same manner as the one in part I with the solution of an example to illustrate the general theory. Part III considers the solution of part of the problem in part II, as $n\to-\infty$ instead of $+\infty$. The last part of the paper gives the solution of $c_x u_{x+3}+b_x u_{x+2}+u_{x+1}+a_x u_x=0$. The method is quite general and is applied to equations of order n. (Received March 14, 1931.)

222. Professor J. V. Uspensky: *A problem in the geometry of numbers.*

A manifold Γ of $n-s$ dimensions is determined in a space of n dimensions by a set of s linear (independent) equations $\sum_{j=1}^{n} c_{ij}x_j=d_i$, $(i=1, 2, 3, \cdots, s)$. If d_i runs independently over all integral values the manifolds Γ form a discrete set. The following question was raised by Professor Harald Bohr in connection with a certain problem in the theory of Diophantine approximations: What upper limit can be set for a minimum distance of a given point from the system of manifolds Γ? In the present paper an answer is given to this question. (Received March 23, 1931.)

223. Mr. W. M. Rust, Jr.: *Integral equations and the cooling problem.*

The first part of this paper deals with a method of solving the generalised Abel integral equation $\int_0^t [G(t, t')u(t')/(t-t')^{\lambda}]dt'=f(t)$, $0<\lambda<1$, for a certain type of absolutely continuous function $G(t, t')$. The function $f(t)$ is *not* assumed to vanish at $t=0$. A Volterra integral equation of the second kind is obtained that has an absolutely continuous solution whose derivative satisfies the preceding equation nearly everywhere.

The remainder of the paper applies this method to the problem of the linear flow of heat in a body composed of several materials. The boundary conditions are the usual ones, as given by Riemann-Weber, *Differentialgleichungen der Physik*, 1912, vol. 2, p. 85. A uniqueness theorem is obtained for general forms of solution, and such solutions are obtained in terms of rapidly converging series of integrals. (Received April 5, 1931.)

224. Dr. R. P. Agnew: *On complex methods of summability.*

This paper raises and discusses the following and related questions: If a method of summability evaluates a complex sequence $\{u_n+iv_n\}$ to $u+iv$, does that method evaluate $\{u_n\}$ to u? (Received April 13, 1931.)

225. Dr. A. B. Brown: *Topological invariance of sub-complexes of singularities.*

We consider the question of topologically invariant sub-sets of a given complex. In the first part of the paper we consider sub-complexes which are determined from the incidence relations by simply counting, and here obtain results of which the following is a simple example: If a complex K is an n-cycle (mod 2), then so is every complex homeomorphic to K. In the second part of the paper, we consider sub-sets (which are sub-complexes) defined in simple terms, but such that it is not known at present how to determine them in all cases from the incidence relations. Given two homeomorphic complexes K and K', in terms of these sub-sets we state a necessary and sufficient condition that the image of a given point of K be on a cell of K' of dimension not exceeding i, under all possible homeomorphisms between K and K'. The proofs depend on the Brouwer theorem of invariance of regionality. (Received April 13, 1931.)

226. Professor V. G. Grove: *The transformation C of nets in hyperspace.*

In this paper the author extends the notion of the transformation C of nets in ordinary space to nets in spaces of dimensions greater than three. An extension is made of the notion of lines in Green's relation R with respect to nets in ordinary space to nets in hyperspace. As in the three dimensional case there is a close relation between the transformation C of nets and the relation R between lines with respect to the net. The derived lines of the congruence G associated with the transformation C intersect the tangents to the curves of the nets in certain covariant points found by Bompiani. Conversely, having given a net possessing these covariant points, all C transforms of the net may be determined by making use of the relation R. (Received April 13, 1931.)

227. Dr. R. S. Underwood: *On universal quadratic null forms in five variables.*

By a transformation due to L. E. Dickson we may reduce the general quadratic null form to the form $F=2^e gaxy+gby^2+cyz+gd\psi(z, w, \cdots)$ where g and a are odd, a is prime to d, c is prime to g, and the greatest common divisor of the coefficients of ψ is 1. In the case of five variables, $\psi=\alpha(hz^2+jzw+lw^2)+Azv+Bwv+Cv^2$. We then apply the following lemma: If each of the congruences: (1) $F\equiv G \pmod{ga}$, and (2), $F\equiv G \pmod{2^e}$, where G is an arbitrary integer, has a solution such that y is odd, then $F=G$ is solvable. This paper derives the conditions necessary and sufficient for the solvability of (1) in the five variable case, and also discusses the briefer results for (2). Auxiliary results are proved which allow greater freedom in the choice of y than was required for the complete solution of the three and four variable cases. (Received April 25, 1931.)

228. Mr. L. B. Robinson: *On a lacuna in the theory of systems of partial differential equations. Part II.*

In a previous notice in this Bulletin, the author considered the system (A): $\partial u/\partial x_i=f_i(x_1, x_2, x_3; u)$, $(i=1, 2, 3)$, and investigated relations connecting the

independent variables when a common solution exists. He assumed that $f_2 \neq 0$, $f_3 \neq 0$ in the neighborhood of the initial conditions. This restriction is needless. Consider two equations (B): $\partial u/\partial x_i = f_i(x_1, x_2; u)$, $(i=1, 2)$. The initial conditions are $u = x_i = 0$, $f_i(0, 0, 0) = 0$. A common solution exists when a relation to be calculated exists between the x_i. $x_2 = \eta(x_1)$, $x_1 = \eta_{-1}(x_2)$. Calculate the solution of the first equation of (B) by successive approximations. Solve the second equation by the same method. To assure equality of the two solutions, solve an integro-differential equation by a convergent series. To demonstrate convergence, consider (1): $\mu x Y + \lambda \int Y dx = x^2 P(xY) + Y^2 Q(xY)$. If certain inequalities are satisfied, the above has a converging solution. It is easy to construct a series like (1) available as a dominant series to prove that the integro-differential equation in question can be solved by a converging series. Extension of the above results to three or more equations is easy. (Received April 27, 1931.)

229. Dr. Gordon Pall (National Research Fellow): *On representation by positive binary quadratic forms.*

In this paper a method is developed of finding a simple closed formula for the number of representations of n in *any* positive integral binary quadratic form *whatever*. Let $f_1, \cdots, f_h$ denote a representative system, consisting of one form from each class, of positive, primitive, integral, binary, quadratic forms of a given discriminant $d < -4$. Let $f_i(n)$ denote the number of representations in f_i of n. The function $r(n) = \frac{1}{2}\{f_1(n) + \cdots + f_h(n)\}$, $(n = 0, \pm 1, \pm 2, \cdots)$, is factorable and well-known, if n is prime to d, and its value being tabulated in the present paper, incidentally, for general n. While not generally factorable, the individual functions $f_i(n)$ satisfy reduction formulae for $f_i(p^\alpha m)$, which are discovered and established here, the type of formula depending only on which of the forms f_j represents p. Combining these formulae gives noteworthy expressions for the number of representations in each f_i. Many beautiful relationships are found among the $f_i(n)$. These results give a concrete expression to the theory of composition. A fuller abstract of these results will appear very soon in the Proceedings of the National Academy of Sciences. (Received May 4, 1931.)

230. Dr. Emma Whiton McDonald: *Magic cubes which are uniform step cubes.*

D. N. Lehmer has applied the theory of congruences to the study of magic squares. In the present paper this theory has been extended to the subject of magic cubes which are uniform step cubes. The magic squares contained in any cube are examined and criteria developed for the determination of the nature of these squares. (Received May 6, 1931.)

231. Professor A. F. Carpenter: *A triad of ruled surfaces defined by reciprocal polars.*

The flecnode curve C of a ruled surface R cuts each line element l of R in two points. The osculating planes of C at pairs of points so determined intersect in lines whose totality constitutes a second ruled surface. For that line of this surface which corresponds to l there is determined its polar reciprocal with respect to the linear complex osculating R along l. The totality of these lines

constitutes a third ruled surface. The paper is concerned with an investigation of the properties of this triad of surfaces. (Received May 8, 1931.)

232. Dr. D. H. Lehmer (National Research Fellow): *A ternary analogue of an abelian group.*

We study in this paper the fundamental properties of a class K of abstract marks subjected to a ternary operation which may be written $a \cdot b \cdot c$. If two postulates (similar to those of Hurwitz for an Abelian group) hold, the class K is called a triplex under the operation. If it is possible to interpret $a \cdot b$ as an element of K then the triplex becomes an ordinary Abelian group under $\cdot$, and is called the extension of the group. The extension of every Abelian group is a triplex, but some triplexes are not the extension of any group. A triplex in general has no unit element. Instead there is associated with each element a, an element a' for which $a \cdot a' \cdot b = b$ for every b. If there is an element e for which $e' = e$, then the triplex is an extension of an Abelian group whose unit is e and conversely. In spite of this lack of a unit, the theory of triplexes of finite order parallels closely that of abelian groups. Precise analogues exist for the theorems of Lagrange and Cayley on subgroups but not for Sylow's theorem. Although every triplex has a set of generators it need not have a basis, or a subtriplex. Simple examples of triplexes such as the set of all odd integers under addition of the set $(i, -1)$ under multiplication, are readily constructed. (Received May 9, 1931.)

233. Professor Dunham Jackson: *Note on the application of Markoff's theorem to problems of approximation in the complex domain.*

In a recent note (this Bulletin, vol. 36 (1930, p. 851) the author has discussed an extension to the complex domain of a method previously used in connection with problems of the approximate representation of real functions, in which the proof of convergence is based on Bernstein's theorem on the derivative of a polynomial or trigonometric sum. The discussion for the case of a complex variable involved certain restrictions on the boundary of the region with which the problem was concerned. The object of the present paper is to show how these restrictions may be somewhat lightened, at the expense, to be sure, of a compensating increase in the stringency of the hypotheses on the function to be approximated, by the use of Markoff's theorem on the derivative of a polynomial in place of that of Bernstein. (Received April 30, 1931.)

234. Professor A. A. Albert: *On direct products.*

The principal contribution of this paper is a set of theorems on the direct product of a normal division algebra and an algebraic field over the same reference field, with applications of the Galois theory of equations. In particular it is shown that it is possible to extend the reference field of a normal division algebra of order p^2, p a prime, such that the algebra over the extended field is a cyclic (Dickson) normal division algebra over this new field. These results are applied to give an independent new proof of a little known theorem of R. Brauer which reduces the problem of determining all normal division algebras of order n^2 to the case where n is a power of a prime. (Received May 6, 1931.)

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SEATTLE, WASHINGTON, June 13, 1931.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, Calif., not later than May 16. For publication in advance of the meeting, abstracts must be received not later than May 9.

MINNEAPOLIS, MINNESOTA, SUMMER MEETING AND COLLOQUIUM, September 8-11, 1931.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wis., not later than August 10. For publication in advance of the meeting, abstracts must be received not later than July 7. Professor Marston Morse will deliver a series of colloquium lectures on *The calculus of variations*. Professor C. C. MacDuffee will deliver an address entitled *Ideals in the theory of linear algebras*.

NEW YORK CITY, October 31, 1931.

Abstracts must be in the hands of Associate Secretary Tomlinson Fort, 501 West 116th Street, New York City, not later than October 10. In order to be printed in the Bulletin in advance of the meeting, abstracts must be in the hands of the Associate Secretary not later than September 7. By invitation of the program committee, Professor D. V. Widder will deliver an address on *Some recent developments in the theory of Laplace integrals*.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

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Changes of address of members should be communicated at once to the Secretary of the Society, R. G. D. Richardson, 501 West 116th St. New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, October, 1930, p. 58), are payable to the Treasurer of the Society, G. W. Mullins, 501 West 116th St., New York.

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